



UNIVERZITET U NOVOM SADU
FAKULTET TEHNIČKIH NAUKA
DEPARTMAN ZA GRAĐEVINARSTVO I GEODEZIJU



***ELABORAT IZ PREDMETA
INŽENJERSKA GEODEZIJA 2***

Jun, 2013. godine

OVERA:

Broj	Napomena:	Overa:
1.		
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3.		
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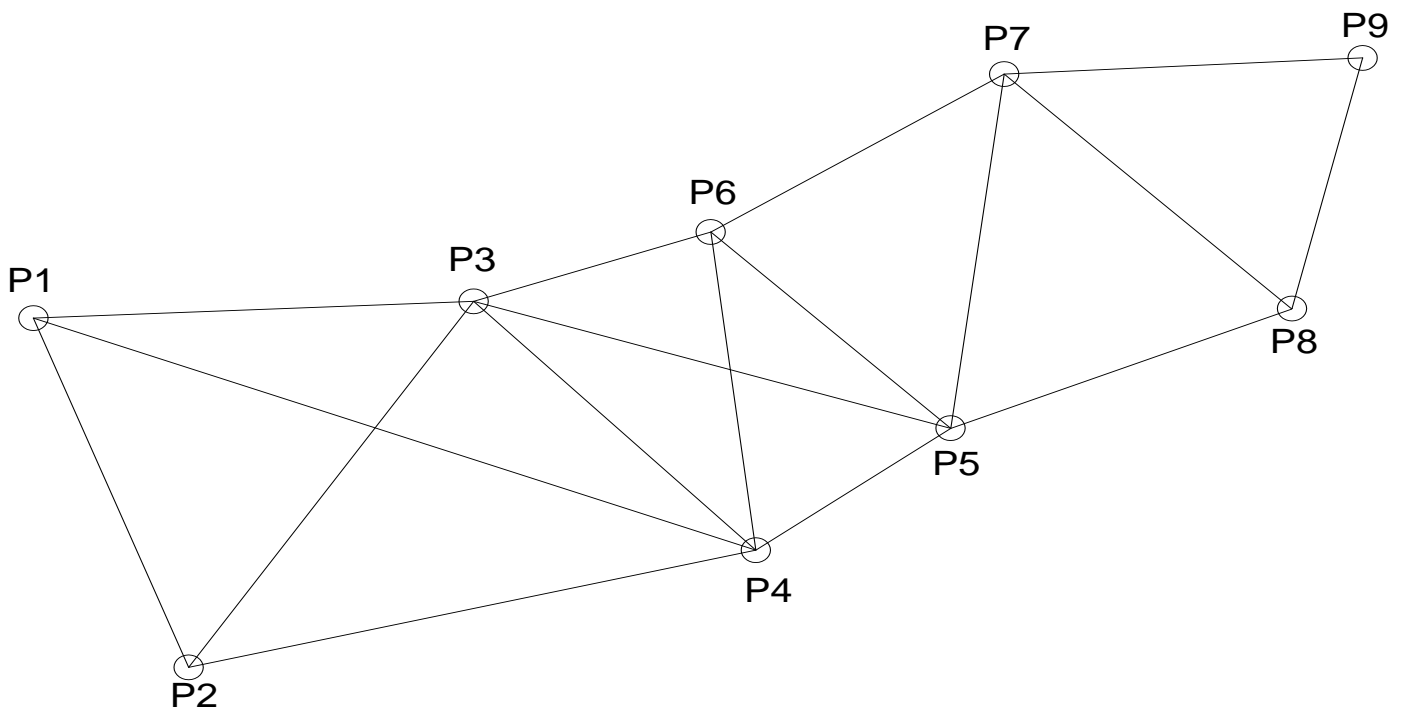
ZADATAK 1

a)

•Plan opažanja

Plan opažanja			
Stanica	Vizura	Pravac	Dužina
P1	P3	X	X
	P4	X	X
	P2	X	X
P2	P1	X	X
	P3	X	X
	P4	X	/
P3	P6	X	/
	P5	X	/
	P4	X	/
	P2	X	X
	P1	X	X
P4	P2	X	X
	P1	X	/
	P3	X	X
	P6	X	X
	P5	X	X
P5	P3	X	/
	P7	X	X
	P8	X	/
P6	P7	X	X
	P5	X	X
P7	P6	X	/
	P9	X	X
	P8	X	X
P8	P5	X	X
	P7	X	X
	P9	X	X

•Skica mreže



• Jednačine popravaka za pravce

$$v_{\alpha_{p1-p3}} = a_{p1p3} \Delta x_{p1p3} + b_{p1p3} \Delta y_{p1p3} + c_{p1p3} \Delta z_1 + f_{\alpha_{p1-p3}}$$

$$v_{\alpha_{p1-p4}} = a_{p1p4} \Delta x_{p1p4} + b_{p1p4} \Delta y_{p1p4} + c_{p1p4} \Delta z_1 + f_{\alpha_{p1-p4}}$$

$$v_{\alpha_{p1-p2}} = a_{p1p2} \Delta x_{p1p2} + b_{p1p2} \Delta y_{p1p2} + c_{p1p2} \Delta z_1 + f_{\alpha_{p1-p2}}$$

$$v_{\alpha_{p2-p1}} = a_{p2p1} \Delta x_{p2p1} + b_{p2p1} \Delta y_{p2p1} + c_{p2p1} \Delta z_2 + f_{\alpha_{p2-p1}}$$

$$v_{\alpha_{p2-p3}} = a_{p2p3} \Delta x_{p2p3} + b_{p2p3} \Delta y_{p2p3} + c_{p2p3} \Delta z_2 + f_{\alpha_{p2-p3}}$$

$$v_{\alpha_{p2-p4}} = a_{p2p4} \Delta x_{p2p4} + b_{p2p4} \Delta y_{p2p4} + c_{p2p4} \Delta z_2 + f_{\alpha_{p2-p4}}$$

$$v_{\alpha_{p3-p6}} = a_{p3p6} \Delta x_{p3p6} + b_{p3p6} \Delta y_{p3p6} + c_{p3p6} \Delta z_3 + f_{\alpha_{p3-p6}}$$

$$v_{\alpha_{p3-p5}} = a_{p3p5} \Delta x_{p3p5} + b_{p3p5} \Delta y_{p3p5} + c_{p3p5} \Delta z_3 + f_{\alpha_{p3-p5}}$$

$$v_{\alpha_{p3-p4}} = a_{p3p4} \Delta x_{p3p4} + b_{p3p4} \Delta y_{p3p4} + c_{p3p4} \Delta z_3 + f_{\alpha_{p3-p4}}$$

$$v_{\alpha_{p3-p2}} = a_{p3p2} \Delta x_{p3p2} + b_{p3p2} \Delta y_{p3p2} + c_{p3p2} \Delta z_3 + f_{\alpha_{p3-p2}}$$

$$v_{\alpha_{p3-p1}} = a_{p3p1} \Delta x_{p3p1} + b_{p3p1} \Delta y_{p3p1} + c_{p3p1} \Delta z_3 + f_{\alpha_{p3-p1}}$$

$$v_{\alpha_{p4-p2}} = a_{p4p2} \Delta x_{p4p2} + b_{p4p2} \Delta y_{p4p2} + c_{p4p2} \Delta z_4 + f_{\alpha_{p4-p2}}$$

$$v_{\alpha_{p4-p1}} = a_{p4p1} \Delta x_{p4p1} + b_{p4p1} \Delta y_{p4p1} + c_{p4p1} \Delta z_4 + f_{\alpha_{p4-p1}}$$

$$v_{\alpha_{p4-p3}} = a_{p4p3} \Delta x_{p4p3} + b_{p4p3} \Delta y_{p4p3} + c_{p4p3} \Delta z_4 + f_{\alpha_{p4-p3}}$$

$$v_{\alpha_{p4-p6}} = a_{p4p6} \Delta x_{p4p6} + b_{p4p6} \Delta y_{p4p6} + c_{p4p6} \Delta z_4 + f_{\alpha_{p4-p6}}$$

$$v_{\alpha_{p4-p5}} = a_{p4p5} \Delta x_{p4p5} + b_{p4p5} \Delta y_{p4p5} + c_{p4p5} \Delta z_4 + f_{\alpha_{p4-p5}}$$

$$v_{\alpha_{p5-p3}} = a_{p5p3} \Delta x_{p5p3} + b_{p5p3} \Delta y_{p5p3} + c_{p5p3} \Delta z_5 + f_{\alpha_{p5-p3}}$$

$$v_{\alpha_{p5-p7}} = a_{p5p7} \Delta x_{p5p7} + b_{p5p7} \Delta y_{p5p7} + c_{p5p7} \Delta z_5 + f_{\alpha_{p5-p7}}$$

$$v_{\alpha_{p5-p8}} = a_{p5p8} \Delta x_{p5p8} + b_{p5p8} \Delta y_{p5p8} + c_{p5p8} \Delta z_5 + f_{\alpha_{p5-p8}}$$

$$v_{\alpha_{p6-p7}} = a_{p6p7} \Delta x_{p6p7} + b_{p6p7} \Delta y_{p6p7} + c_{p6p7} \Delta z_6 + f_{\alpha_{p6-p7}}$$

$$v_{\alpha_{p6-p5}} = a_{p6p5} \Delta x_{p6p5} + b_{p6p5} \Delta y_{p6p5} + c_{p6p5} \Delta z_6 + f_{\alpha_{p6-p5}}$$

$$v_{\alpha_{p7-p6}} = a_{p7p6} \Delta x_{p7p6} + b_{p7p6} \Delta y_{p7p6} + c_{p7p6} \Delta z_7 + f_{\alpha_{p7-p6}}$$

$$v_{\alpha_{p7-p9}} = a_{p7p9} \Delta x_{p7p9} + b_{p7p9} \Delta y_{p7p9} + c_{p7p9} \Delta z_7 + f_{\alpha_{p7-p9}}$$

$$v_{\alpha_{p7-p8}} = a_{p7p8} \Delta x_{p7p8} + b_{p7p8} \Delta y_{p7p8} + c_{p7p8} \Delta z_7 + f_{\alpha_{p7-p8}}$$

$$v_{\alpha_{p8-p5}} = a_{p8p5} \Delta x_{p8p5} + b_{p8p5} \Delta y_{p8p5} + c_{p8p5} \Delta z_8 + f_{\alpha_{p8-p5}}$$

$$v_{\alpha_{p8-p7}} = a_{p8p7} \Delta x_{p8p7} + b_{p8p7} \Delta y_{p8p7} + c_{p8p7} \Delta z_8 + f_{\alpha_{p8-p7}}$$

$$v_{\alpha_{p8-p9}} = a_{p8p9} \Delta x_{p8p9} + b_{p8p9} \Delta y_{p8p9} + c_{p8p9} \Delta z_8 + f_{\alpha_{p8-p9}}$$

$$a_{ij} = \frac{\rho'' \sin v_i^j}{D_{i-j}^0} \quad b_{ij} = -\frac{\rho'' \cos v_i^j}{D_{i-j}^0} \quad c_{ij} = 1$$

• Jednačine popravaka za dužine

$$v_{d_{p1-p3}} = A_{p1p3} \Delta x_{p1p3} + B_{p1p3} \Delta y_{p1p3} + f_{d_{p1-p3}}$$

$$v_{d_{p1-p4}} = A_{p1p4} \Delta x_{p1p4} + B_{p1p4} \Delta y_{p1p4} + f_{d_{p1-p4}}$$

$$v_{d_{p1-p2}} = A_{p1p2} \Delta x_{p1p2} + B_{p1p2} \Delta y_{p1p2} + f_{d_{p1-p2}}$$

$$v_{d_{p2-p1}} = A_{p2p1} \Delta x_{p2p1} + B_{p2p1} \Delta y_{p2p1} + f_{d_{p2-p1}}$$

$$v_{d_{p2-p3}} = A_{p2p3} \Delta x_{p2p3} + B_{p2p3} \Delta y_{p2p3} + f_{d_{p2-p3}}$$

$$v_{d_{p3-p2}} = A_{p3p2} \Delta x_{p3p2} + B_{p3p2} \Delta y_{p3p2} + f_{d_{p3-p2}}$$

$$v_{d_{p3-p1}} = A_{p3p1} \Delta x_{p3p1} + B_{p3p1} \Delta y_{p3p1} + f_{d_{p3-p1}}$$

$$v_{d_{p4-p2}} = A_{p4p2} \Delta x_{p4p2} + B_{p4p2} \Delta y_{p4p2} + f_{d_{p4-p2}}$$

$$v_{d_{p4-p3}} = A_{p4p3} \Delta x_{p4p3} + B_{p4p3} \Delta y_{p4p3} + f_{d_{p4-p3}}$$

$$v_{d_{p4-p6}} = A_{p4p6} \Delta x_{p4p6} + B_{p4p6} \Delta y_{p4p6} + f_{d_{p4-p6}}$$

$$v_{d_{p4-p5}} = A_{p4p5} \Delta x_{p4p5} + B_{p4p5} \Delta y_{p4p5} + f_{d_{p4-p5}}$$

$$v_{d_{p5-p7}} = A_{p5p7} \Delta x_{p5p7} + B_{p5p7} \Delta y_{p5p7} + f_{d_{p5-p7}}$$

$$v_{d_{p6-p7}} = A_{p6p7} \Delta x_{p6p7} + B_{p6p7} \Delta y_{p6p7} + f_{d_{p6-p7}}$$

$$v_{d_{p6-p5}} = A_{p6p5} \Delta x_{p6p5} + B_{p6p5} \Delta y_{p6p5} + f_{d_{p6-p5}}$$

$$v_{d_{p7-p9}} = A_{p7p9} \Delta x_{p7p9} + B_{p7p9} \Delta y_{p7p9} + f_{d_{p7-p9}}$$

$$v_{d_{p7-p8}} = A_{p7p8} \Delta x_{p7p8} + B_{p7p8} \Delta y_{p7p8} + f_{d_{p7-p8}}$$

$$v_{d_{p8-p5}} = A_{p8p5} \Delta x_{p8p5} + B_{p8p5} \Delta y_{p8p5} + f_{d_{p8-p5}}$$

$$v_{d_{p8-p7}} = A_{p8p7} \Delta x_{p8p7} + B_{p8p7} \Delta y_{p8p7} + f_{d_{p8-p7}}$$

$$v_{d_{p8-p9}} = A_{p8p9} \Delta x_{p8p9} + B_{p8p9} \Delta y_{p8p9} + f_{d_{p8-p9}}$$

$$A_{ij} = -\sin \nu_i^j \quad B_{ij} = -\cos \nu_i^j$$

•Homogenizacija težina

Standardno odstupanje pravca je $\sigma_{\alpha_{i-j}} = 1''$.

Pravci su mereni u dva girusa. Dužine su merene u dva ponavljanja.

$$\sigma_{\alpha_i} = \frac{\sigma_{\alpha_{i-j}}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071''$$

$$P_{\alpha_{i-j}} = \frac{\sigma_0^2}{\sigma_{\alpha_{i-j}}^2} = \frac{c}{\sigma_{\alpha_{i-j}}^2}, \quad \sigma_0^2 = 0.7071^2 = 0.5 \quad P_{d_{i-j}} = \frac{\sigma_0^2}{\sigma_{d_{i-j}}^2} = \frac{c}{\sigma_{d_{i-j}}^2}, \quad \sigma_{d_{i-j}} = \frac{1mm+1ppm}{\sqrt{2}}$$

$$P_{\alpha_{i-j}} = 1$$

$$\sigma_{d_{p1-p3}} = 0.8504 \text{ mm}$$

$$\sigma_{d_{p5-p7}} = 0.8439 \text{ mm}$$

$$\sigma_{d_{p1-p4}} = 0.9582 \text{ mm}$$

$$\sigma_{d_{p6-p7}} = 0.8201 \text{ mm}$$

$$\sigma_{d_{p1-p2}} = 0.8502 \text{ mm}$$

$$\sigma_{d_{p6-p5}} = 0.8154 \text{ mm}$$

$$\sigma_{d_{p2-p1}} = 0.8502 \text{ mm}$$

$$\sigma_{d_{p7-p9}} = 0.8238 \text{ mm}$$

$$\sigma_{d_{p2-p3}} = 0.8752 \text{ mm}$$

$$\sigma_{d_{p7-p8}} = 0.8368 \text{ mm}$$

$$\sigma_{d_{p3-p2}} = 0.8752 \text{ mm}$$

$$\sigma_{d_{p8-p5}} = 0.8272 \text{ mm}$$

$$\sigma_{d_{p3-p1}} = 0.8504 \text{ mm}$$

$$\sigma_{d_{p8-p7}} = 0.8368 \text{ mm}$$

$$\sigma_{d_{p4-p2}} = 0.8969 \text{ mm}$$

$$\sigma_{d_{p8-p9}} = 0.8059 \text{ mm}$$

$$\sigma_{d_{p4-p3}} = 0.8394 \text{ mm}$$

$$\sigma_{d_{p4-p5}} = 0.7858 \text{ mm}$$

$$\sigma_{d_{p4-p6}} = 0.8299 \text{ mm}$$

•Standardno odstupanje nepozatih parametara

$$m_{xpi} = \sigma_0 \cdot \sqrt{Q_{x_ix_i}}$$

$$m_{ypi} = \sigma_0 \cdot \sqrt{Q_{y_iy_i}}$$

$$m_{zi} = \sigma_0 \cdot \sqrt{Q_{z_iz_i}}$$

$$m_{xp1} = 2.0576 \text{ mm}$$

$$m_{yp5} = 0.5150 \text{ mm}$$

$$m_{z1} = 0,8995 \text{ "}$$

$$m_{yp1} = 0.7611 \text{ mm}$$

$$m_{xp6} = 1.1436 \text{ mm}$$

$$m_{z2} = 0.8630 \text{ "}$$

$$m_{xp2} = 1.8205 \text{ mm}$$

$$m_{yp6} = 0.5967 \text{ mm}$$

$$m_{z3} = 0.8198 \text{ "}$$

$$m_{yp2} = 0.6460 \text{ mm}$$

$$m_{xp7} = 0.7331 \text{ mm}$$

$$m_{z4} = 0.7523 \text{ "}$$

$$m_{xp3} = 1.4129 \text{ mm}$$

$$m_{yp7} = 0.5753 \text{ mm}$$

$$m_{z5} = 0.8422 \text{ "}$$

$$m_{yp3} = 0.7180 \text{ mm}$$

$$m_{xp8} = 0.0000 \text{ mm}$$

$$m_{z6} = 0.9386 \text{ "}$$

$$m_{xp4} = 1.1089 \text{ mm}$$

$$m_{yp8} = 0.0000 \text{ mm}$$

$$m_{z7} = 0.7633 \text{ "}$$

$$m_{yp4} = 0.4788 \text{ mm}$$

$$m_{xp9} = 0.0000 \text{ mm}$$

$$m_{z8} = 0.9205 \text{ "}$$

$$m_{xp5} = 0.7811 \text{ mm}$$

$$m_{yp9} = 0.6071 \text{ mm}$$

•Standardno odstupanje položaja tačke

$$m_{pi} = \sqrt{m_{xpi}^2 + m_{ypi}^2}$$

$$m_{p1} = 2.1938 \text{ mm}$$

$$m_{p2} = 1.9317 \text{ mm}$$

$$m_{p3} = 1.5849 \text{ mm}$$

$$m_{p4} = 1.2079 \text{ mm}$$

$$m_{p5} = 0.9356 \text{ mm}$$

$$m_{p6} = 1.2899 \text{ mm}$$

$$m_{p7} = 0.9319 \text{ mm}$$

$$m_{p8} = 0.0000 \text{ mm}$$

$$m_{p9} = 0.6071 \text{ mm}$$

•Elementi elipse grešaka

$$k = \sqrt{(Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2} \quad \lambda_1 = \frac{1}{2}(Q_{xx} + Q_{yy} + k) \quad \lambda_2 = \frac{1}{2}(Q_{xx} + Q_{yy} - k)$$

$$c = \sigma_0 \sqrt{\chi_{1-\alpha,2}^2} \quad A = c\sqrt{\lambda_1} \quad B = c\sqrt{\lambda_2} \quad \operatorname{tg} 2\theta = \frac{2Q_{xy}}{Q_{xx} - Q_{yy}}$$

A – velika poluosa elipse, B – mala poluosa elipse,

θ – ugao koji velika poluosa A zaklapa sa pravcem severa

$A_{P1} = 5.1427 \text{ mm}$	$B_{P1} = 1.5457 \text{ mm}$	$\theta_{P1} = 12^\circ 14' 24''$	$\frac{A_{P1}}{B_{P1}} = 3.3271$
$A_{P2} = 4.4691 \text{ mm}$	$B_{P2} = 1.5442 \text{ mm}$	$\theta_{P2} = 355^\circ 20' 26''$	$\frac{A_{P2}}{B_{P2}} = 2.8941$
$A_{P3} = 3.5819 \text{ mm}$	$B_{P3} = 1.4900 \text{ mm}$	$\theta_{P3} = 16^\circ 37' 43''$	$\frac{A_{P3}}{B_{P3}} = 2.4039$
$A_{P4} = 2.7155 \text{ mm}$	$B_{P4} = 1.1692 \text{ mm}$	$\theta_{P4} = 358^\circ 8' 17''$	$\frac{A_{P4}}{B_{P4}} = 2.3225$
$A_{P5} = 1.9185 \text{ mm}$	$B_{P5} = 1.2506 \text{ mm}$	$\theta_{P5} = 353^\circ 43' 36''$	$\frac{A_{P5}}{B_{P5}} = 1.5341$
$A_{P6} = 2.8903 \text{ mm}$	$B_{P6} = 1.2710 \text{ mm}$	$\theta_{P6} = 16^\circ 05' 27''$	$\frac{A_{P6}}{B_{P6}} = 2.2741$
$A_{P7} = 2.0851 \text{ mm}$	$B_{P7} = 0.9249 \text{ mm}$	$\theta_{P7} = 34^\circ 37' 49''$	$\frac{A_{P7}}{B_{P7}} = 2.2544$
$A_{P9} = 1.4861 \text{ mm}$	$B_{P9} = 0.0000 \text{ mm}$	$\theta_{P9} = 00^\circ 00' 00''$	

•Matrica kofaktora popravaka Q_V $Q_V = P^{-1} - AQ_x A^T$

Q_{Vdiag}

$$= \begin{bmatrix} 0.5272 & 0.5735 & 0.4262 & 0.4298 & 0.5615 & 0.4812 & 0.2055 & 0.5226 & 0.5026 & 0.4720 \\ 0.4293 & 0.5140 & 0.6162 & 0.5125 & 0.3328 & 0.1960 & 0.2269 & 0.3813 & 0.3200 & 0.2473 \\ 0.2473 & 0.2603 & 0.1367 & 0.2419 & 0.3714 & 0.4219 & 0.2414 & 1.0356 & 1.1846 & 1.0151 \\ 1.0151 & 1.1400 & 1.1400 & 1.0356 & 0.1524 & 0.9750 & 0.9054 & 0.6250 & 1.0788 & 0.6664 \\ 0.9602 & 0.8229 & 1.0852 & 0.7917 & 1.0852 & 1.259] \end{bmatrix}$$

- Koefficienti unutrašnje pouzdanosti r_i

$$r_i = P_{ii} Q_{v_{ii}} \quad r_i \geq 0.30 \Rightarrow \text{Merenje je pouzdano}$$

$r_1 = 0.5272$	$r_{24} = \mathbf{0.2419}$
$r_2 = 0.5735$	$r_{25} = 0.3714$
$r_3 = 0.4262$	$r_{26} = 0.4219$
$r_4 = 0.4298$	$r_{27} = \mathbf{0.2414}$
$r_5 = 0.5615$	$r_{28} = 0.7161$
$r_6 = 0.4812$	$r_{29} = 0.6451$
$r_7 = \mathbf{0.2055}$	$r_{30} = 0.7022$
$r_8 = 0.5226$	$r_{31} = 0.7022$
$r_9 = 0.5026$	$r_{32} = 0.7441$
$r_{10} = 0.4720$	$r_{33} = 0.7441$
$r_{11} = 0.4293$	$r_{34} = 0.7161$
$r_{12} = 0.5140$	$r_{35} = \mathbf{0.0947}$
$r_{13} = 0.6162$	$r_{36} = 0.6919$
$r_{14} = 0.5125$	$r_{37} = 0.6573$
$r_{15} = 0.3328$	$r_{38} = 0.5061$
$r_{16} = \mathbf{0.1960}$	$r_{39} = 0.7574$
$r_{17} = \mathbf{0.2269}$	$r_{40} = 0.4954$
$r_{18} = 0.3813$	$r_{41} = 0.7221$
$r_{19} = 0.3224$	$r_{42} = 0.6063$
$r_{20} = \mathbf{0.2473}$	$r_{43} = 0.7748$
$r_{21} = \mathbf{0.2473}$	$r_{44} = 0.5785$
$r_{22} = \mathbf{0.2603}$	$r_{45} = 0.7748$
$r_{23} = \mathbf{0.1367}$	$r_{46} = 0.9694$

•Marginalna gruba greška G_i koja se može otkriti *Data-snooping* testom

$$|G_i| = \frac{\sigma_0 \sqrt{\lambda_0}}{P_i \sqrt{Q_{v_{ii}}}} \quad |G_i| < 7\sigma_0 \quad 7\sigma_0 = 4,9497$$

$$\lambda_0 = t_{1-\beta_0} + t_{1-\alpha/2} = 0,842 + 1,96 = 2,802$$

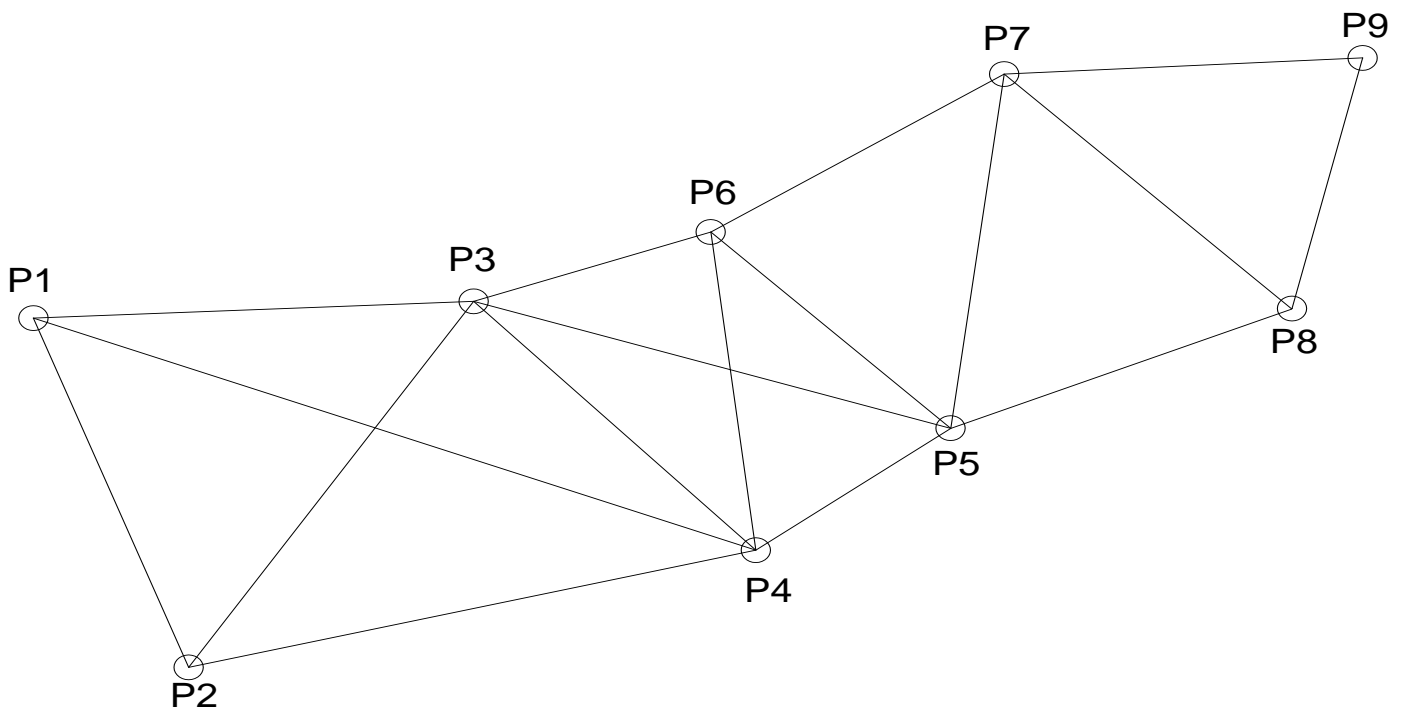
$G_1 = 1.4417$	$G_{17} = 0.9458$	$G_{32} = 3.2479$
$G_2 = 1.5037$	$G_{18} = 1.2261$	$G_{33} = 3.2479$
$G_3 = 1.2963$	$G_{19} = 1.1274$	$G_{34} = 2.9222$
$G_4 = 1.3017$	$G_{20} = 0.9874$	$G_{35} = 1.2471$
$G_5 = 1.4878$	$G_{21} = 0.9874$	$G_{36} = 2.7630$
$G_6 = 1.3774$	$G_{22} = 1.0130$	$G_{37} = 2.6026$
$G_7 = 0.9001$	$G_{23} = 0.7341$	$G_{38} = 1.9386$
$G_8 = 1.4354$	$G_{24} = 0.9766$	$G_{39} = 2.9375$
$G_9 = 1.4076$	$G_{25} = 1.2100$	$G_{40} = 2.1802$
$G_{10} = 1.3641$	$G_{26} = 1.2897$	$G_{41} = 2.5874$
$G_{11} = 1.3010$	$G_{27} = 0.9756$	$G_{42} = 2.4447$
$G_{12} = 1.4235$	$G_{28} = 2.9222$	$G_{43} = 2.8970$
$G_{13} = 1.5586$	$G_{29} = 3.9686$	$G_{44} = 2.4178$
$G_{14} = 1.4214$	$G_{30} = 2.8918$	$G_{45} = 2.8970$
$G_{15} = 1.1454$	$G_{31} = 2.8918$	$G_{46} = 2.8936$
$G_{16} = 0.8790$		

b)

•Plan opažanja

Plan opažanja			
Stanica	Vizura	Pravac	Dužina
P1	P3	X	X
	P4	X	X
	P2	X	X
P2	P1	X	X
	P3	X	X
	P4	X	/
P3	P6	X	/
	P5	X	/
	P4	X	/
	P2	X	X
	P1	X	X
P4	P2	X	X
	P1	X	/
	P3	X	X
	P6	X	X
	P5	X	X
P5	P3	X	/
	P7	X	X
	P8	X	/
P6	P7	X	X
	P5	X	X
P7	P6	X	/
	P9	X	X
	P8	X	X
P8	P5	X	X
	P7	X	X
	P9	X	X

•Skica mreže



• Jednačine popravaka za pravce

$$v_{\alpha_{p1-p3}} = a_{p1p3} \Delta x_{p1p3} + b_{p1p3} \Delta y_{p1p3} + c_{p1p3} \Delta z_1 + f_{\alpha_{p1-p3}}$$

$$v_{\alpha_{p1-p4}} = a_{p1p4} \Delta x_{p1p4} + b_{p1p4} \Delta y_{p1p4} + c_{p1p4} \Delta z_1 + f_{\alpha_{p1-p4}}$$

$$v_{\alpha_{p1-p2}} = a_{p1p2} \Delta x_{p1p2} + b_{p1p2} \Delta y_{p1p2} + c_{p1p2} \Delta z_1 + f_{\alpha_{p1-p2}}$$

$$v_{\alpha_{p2-p1}} = a_{p2p1} \Delta x_{p2p1} + b_{p2p1} \Delta y_{p2p1} + c_{p2p1} \Delta z_2 + f_{\alpha_{p2-p1}}$$

$$v_{\alpha_{p2-p3}} = a_{p2p3} \Delta x_{p2p3} + b_{p2p3} \Delta y_{p2p3} + c_{p2p3} \Delta z_2 + f_{\alpha_{p2-p3}}$$

$$v_{\alpha_{p2-p4}} = a_{p2p4} \Delta x_{p2p4} + b_{p2p4} \Delta y_{p2p4} + c_{p2p4} \Delta z_2 + f_{\alpha_{p2-p4}}$$

$$v_{\alpha_{p3-p6}} = a_{p3p6} \Delta x_{p3p6} + b_{p3p6} \Delta y_{p3p6} + c_{p3p6} \Delta z_3 + f_{\alpha_{p3-p6}}$$

$$v_{\alpha_{p3-p5}} = a_{p3p5} \Delta x_{p3p5} + b_{p3p5} \Delta y_{p3p5} + c_{p3p5} \Delta z_3 + f_{\alpha_{p3-p5}}$$

$$v_{\alpha_{p3-p4}} = a_{p3p4} \Delta x_{p3p4} + b_{p3p4} \Delta y_{p3p4} + c_{p3p4} \Delta z_3 + f_{\alpha_{p3-p4}}$$

$$v_{\alpha_{p3-p2}} = a_{p3p2} \Delta x_{p3p2} + b_{p3p2} \Delta y_{p3p2} + c_{p3p2} \Delta z_3 + f_{\alpha_{p3-p2}}$$

$$v_{\alpha_{p3-p1}} = a_{p3p1} \Delta x_{p3p1} + b_{p3p1} \Delta y_{p3p1} + c_{p3p1} \Delta z_3 + f_{\alpha_{p3-p1}}$$

$$v_{\alpha_{p4-p2}} = a_{p4p2} \Delta x_{p4p2} + b_{p4p2} \Delta y_{p4p2} + c_{p4p2} \Delta z_4 + f_{\alpha_{p4-p2}}$$

$$v_{\alpha_{p4-p1}} = a_{p4p1} \Delta x_{p4p1} + b_{p4p1} \Delta y_{p4p1} + c_{p4p1} \Delta z_4 + f_{\alpha_{p4-p1}}$$

$$v_{\alpha_{p4-p3}} = a_{p4p3} \Delta x_{p4p3} + b_{p4p3} \Delta y_{p4p3} + c_{p4p3} \Delta z_4 + f_{\alpha_{p4-p3}}$$

$$v_{\alpha_{p4-p6}} = a_{p4p6} \Delta x_{p4p6} + b_{p4p6} \Delta y_{p4p6} + c_{p4p6} \Delta z_4 + f_{\alpha_{p4-p6}}$$

$$v_{\alpha_{p4-p5}} = a_{p4p5} \Delta x_{p4p5} + b_{p4p5} \Delta y_{p4p5} + c_{p4p5} \Delta z_4 + f_{\alpha_{p4-p5}}$$

$$v_{\alpha_{p5-p3}} = a_{p5p3} \Delta x_{p5p3} + b_{p5p3} \Delta y_{p5p3} + c_{p5p3} \Delta z_5 + f_{\alpha_{p5-p3}}$$

$$v_{\alpha_{p5-p7}} = a_{p5p7} \Delta x_{p5p7} + b_{p5p7} \Delta y_{p5p7} + c_{p5p7} \Delta z_5 + f_{\alpha_{p5-p7}}$$

$$v_{\alpha_{p5-p8}} = a_{p5p8} \Delta x_{p5p8} + b_{p5p8} \Delta y_{p5p8} + c_{p5p8} \Delta z_5 + f_{\alpha_{p5-p8}}$$

$$v_{\alpha_{p6-p7}} = a_{p6p7} \Delta x_{p6p7} + b_{p6p7} \Delta y_{p6p7} + c_{p6p7} \Delta z_6 + f_{\alpha_{p6-p7}}$$

$$v_{\alpha_{p6-p5}} = a_{p6p5} \Delta x_{p6p5} + b_{p6p5} \Delta y_{p6p5} + c_{p6p5} \Delta z_6 + f_{\alpha_{p6-p5}}$$

$$v_{\alpha_{p7-p6}} = a_{p7p6} \Delta x_{p7p6} + b_{p7p6} \Delta y_{p7p6} + c_{p7p6} \Delta z_7 + f_{\alpha_{p7-p6}}$$

$$v_{\alpha_{p7-p9}} = a_{p7p9} \Delta x_{p7p9} + b_{p7p9} \Delta y_{p7p9} + c_{p7p9} \Delta z_7 + f_{\alpha_{p7-p9}}$$

$$v_{\alpha_{p7-p8}} = a_{p7p8} \Delta x_{p7p8} + b_{p7p8} \Delta y_{p7p8} + c_{p7p8} \Delta z_7 + f_{\alpha_{p7-p8}}$$

$$v_{\alpha_{p8-p5}} = a_{p8p5} \Delta x_{p8p5} + b_{p8p5} \Delta y_{p8p5} + c_{p8p5} \Delta z_8 + f_{\alpha_{p8-p5}}$$

$$v_{\alpha_{p8-p7}} = a_{p8p7} \Delta x_{p8p7} + b_{p8p7} \Delta y_{p8p7} + c_{p8p7} \Delta z_8 + f_{\alpha_{p8-p7}}$$

$$v_{\alpha_{p8-p9}} = a_{p8p9} \Delta x_{p8p9} + b_{p8p9} \Delta y_{p8p9} + c_{p8p9} \Delta z_8 + f_{\alpha_{p8-p9}}$$

$$a_{ij} = \frac{\rho'' \sin v_i^j}{D_{i-j}^0} \quad b_{ij} = -\frac{\rho'' \cos v_i^j}{D_{i-j}^0} \quad c_{ij} = 1$$

• Jednačine popravaka za dužine

$$v_{d_{p1-p3}} = A_{p1p3} \Delta x_{p1p3} + B_{p1p3} \Delta y_{p1p3} + f_{d_{p1-p3}}$$

$$v_{d_{p1-p4}} = A_{p1p4} \Delta x_{p1p4} + B_{p1p4} \Delta y_{p1p4} + f_{d_{p1-p4}}$$

$$v_{d_{p1-p2}} = A_{p1p2} \Delta x_{p1p2} + B_{p1p2} \Delta y_{p1p2} + f_{d_{p1-p2}}$$

$$v_{d_{p2-p1}} = A_{p2p1} \Delta x_{p2p1} + B_{p2p1} \Delta y_{p2p1} + f_{d_{p2-p1}}$$

$$v_{d_{p2-p3}} = A_{p2p3} \Delta x_{p2p3} + B_{p2p3} \Delta y_{p2p3} + f_{d_{p2-p3}}$$

$$v_{d_{p3-p2}} = A_{p3p2} \Delta x_{p3p2} + B_{p3p2} \Delta y_{p3p2} + f_{d_{p3-p2}}$$

$$v_{d_{p3-p1}} = A_{p3p1} \Delta x_{p3p1} + B_{p3p1} \Delta y_{p3p1} + f_{d_{p3-p1}}$$

$$v_{d_{p4-p2}} = A_{p4p2} \Delta x_{p4p2} + B_{p4p2} \Delta y_{p4p2} + f_{d_{p4-p2}}$$

$$v_{d_{p4-p3}} = A_{p4p3} \Delta x_{p4p3} + B_{p4p3} \Delta y_{p4p3} + f_{d_{p4-p3}}$$

$$v_{d_{p4-p6}} = A_{p4p6} \Delta x_{p4p6} + B_{p4p6} \Delta y_{p4p6} + f_{d_{p4-p6}}$$

$$v_{d_{p4-p5}} = A_{p4p5} \Delta x_{p4p5} + B_{p4p5} \Delta y_{p4p5} + f_{d_{p4-p5}}$$

$$v_{d_{p5-p7}} = A_{p5p7} \Delta x_{p5p7} + B_{p5p7} \Delta y_{p5p7} + f_{d_{p5-p7}}$$

$$v_{d_{p6-p7}} = A_{p6p7} \Delta x_{p6p7} + B_{p6p7} \Delta y_{p6p7} + f_{d_{p6-p7}}$$

$$v_{d_{p6-p5}} = A_{p6p5} \Delta x_{p6p5} + B_{p6p5} \Delta y_{p6p5} + f_{d_{p6-p5}}$$

$$v_{d_{p7-p9}} = A_{p7p9} \Delta x_{p7p9} + B_{p7p9} \Delta y_{p7p9} + f_{d_{p7-p9}}$$

$$v_{d_{p7-p8}} = A_{p7p8} \Delta x_{p7p8} + B_{p7p8} \Delta y_{p7p8} + f_{d_{p7-p8}}$$

$$v_{d_{p8-p5}} = A_{p8p5} \Delta x_{p8p5} + B_{p8p5} \Delta y_{p8p5} + f_{d_{p8-p5}}$$

$$v_{d_{p8-p7}} = A_{p8p7} \Delta x_{p8p7} + B_{p8p7} \Delta y_{p8p7} + f_{d_{p8-p7}}$$

$$v_{d_{p8-p9}} = A_{p8p9} \Delta x_{p8p9} + B_{p8p9} \Delta y_{p8p9} + f_{d_{p8-p9}}$$

$$A_{ij} = -\sin \nu_i^j \quad B_{ij} = -\cos \nu_i^j$$

•Homogenizacija težina

Standardno odstupanje pravca je $\sigma_{\alpha_{i-j}} = 1''$.

Pravci su mereni u dva girusa. Dužine su merene u dva ponavljanja.

$$\sigma_{\alpha_i} = \frac{\sigma_{\alpha_{i-j}}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071''$$

$$P_{\alpha_{i-j}} = \frac{\sigma_0^2}{\sigma_{\alpha_{i-j}}^2} = \frac{c}{\sigma_{\alpha_{i-j}}^2}, \quad \sigma_0^2 = 0.7071^2 = 0.5 \quad P_{d_{i-j}} = \frac{\sigma_0^2}{\sigma_{d_{i-j}}^2} = \frac{c}{\sigma_{d_{i-j}}^2}, \quad \sigma_{d_{i-j}} = \frac{1mm+1ppm}{\sqrt{2}}$$

$$P_{\alpha_{i-j}} = 1$$

$$\sigma_{d_{p1-p3}} = 0.8504 \text{ mm}$$

$$\sigma_{d_{p5-p7}} = 0.8439 \text{ mm}$$

$$\sigma_{d_{p1-p4}} = 0.9582 \text{ mm}$$

$$\sigma_{d_{p6-p7}} = 0.8201 \text{ mm}$$

$$\sigma_{d_{p1-p2}} = 0.8502 \text{ mm}$$

$$\sigma_{d_{p6-p5}} = 0.8154 \text{ mm}$$

$$\sigma_{d_{p2-p1}} = 0.8502 \text{ mm}$$

$$\sigma_{d_{p7-p9}} = 0.8238 \text{ mm}$$

$$\sigma_{d_{p2-p3}} = 0.8752 \text{ mm}$$

$$\sigma_{d_{p7-p8}} = 0.8368 \text{ mm}$$

$$\sigma_{d_{p3-p2}} = 0.8752 \text{ mm}$$

$$\sigma_{d_{p8-p5}} = 0.8272 \text{ mm}$$

$$\sigma_{d_{p3-p1}} = 0.8504 \text{ mm}$$

$$\sigma_{d_{p8-p7}} = 0.8368 \text{ mm}$$

$$\sigma_{d_{p4-p2}} = 0.8969 \text{ mm}$$

$$\sigma_{d_{p8-p9}} = 0.8059 \text{ mm}$$

$$\sigma_{d_{p4-p3}} = 0.8394 \text{ mm}$$

$$\sigma_{d_{p4-p5}} = 0.7858 \text{ mm}$$

$$\sigma_{d_{p4-p6}} = 0.8299 \text{ mm}$$

•Matrica dizajna A

	X1	Y1	X2	Y2	X3	Y3	X4	Y4	X5	Y5	X6	Y6	X7	Y7	X8	Y8	X9	Y9	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
	1,017111	-0,04561188	0	0	-1,01711	0,045611884	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0,543159	0,205687647	0	0	0	0	-0,54316	-0,20569	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0,359612	0,954046814	-0,35961	-0,95405	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0,359612	0,954046814	-0,35961	-0,95405	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0,478158	-0,72392	-0,47816	0,723916463	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0,746669	-0,18192	0	0	-0,74667	0,181917	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	1,691358	-0,58358988	0	0	0	0	-1,691358379	0,58359	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0,856612	0,268485924	0	0	-0,856612	-0,26849	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0,76424	0,794354102	-0,76424	-0,79435	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0,478158	-0,72392	-0,47816	0,723916463	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	1,017111	-0,04561188	0	0	-1,01711	0,045611884	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0,746669	-0,18192	0	0	-0,74667	0,181917	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0,543159	0,205687647	0	0	0	0	-0,54316	-0,20569	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0,76424	0,794354102	-0,76424	-0,79435	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	-0,14191	-1,17901	0	0	0,141912895	1,179011	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	1,49072	-1,1012	-1,49072	1,101196	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0,856612	0,268485924	0	0	-0,856612	-0,26849	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0,135826	-1,05743	0	0	-0,13583	1,057427	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1,122508	-0,46358	0	0	0	0	-1,12251	0,463576	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	1,090197883	-0,69145	-1,0902	0,691454	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	-0,969356	-0,93461	0,969355544	0,934607	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	1,090197883	-0,69145	-1,0902	0,691454	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	1,248107	-0,06631	0	0	-1,24811	-1,24811	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0,810971	0,778711	-0,81097	-0,77871	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	1,122508	-0,46358	0	0	0	0	-1,12251	0,463576	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0,810971	0,778711	-0,81097	-0,77871	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,343381	-1,43655	-0,34338	1,436552	0	0	0	0	0	0	0	1
	-0,0448	-0,998996	0	0	0,0448	0,998995997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0,354145	-0,93519038	0	0	0	0	-0,35415	0,93519	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0,935733	-0,35270904	-0,93573	0,352709	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0,935733	-0,35270904	-0,93573	0,352709	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	-0,83441	-0,55114	0,834412	0,551141606	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	-0,83441	-0,55114	0,834412	0,551141606	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	-0,0448	-0,998996	0	0	0,0448	0,998995997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	-0,23671	0,746669	0	0	0,236714	0,971579	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0,720634	-0,69331535	-0,72063	0,693315	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	-0,99283	0,119503	0	0	0,992833785	-0,1195	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	-0,59417	-0,80434	0,594167	0,804341	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	-0,991851	-0,1274	0	0	0,991851	0,127402	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	-0,535602013	-0,84447	0,535602	0,844471	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	-0,694086	0,719892	0,694086157	-0,71989	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	-0,05306	-0,99859	0	0	0,053057	0,721308	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0,692615	-0,72131	-0,69261	0,721308	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	-0,381712	-0,92428	0	0	0	0	0,381712	0,924281	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0,692615	-0,72131	-0,69261	0,721308	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0,9726	-0,23248	0,972601	0,232482	0	0	0	0	0	0	0	0

A =

•Standardno odstupanje nepozatih parametara

$$m_{xpi} = \sigma_0 \cdot \sqrt{Q_{x_i x_i}}$$

$$m_{ypi} = \sigma_0 \cdot \sqrt{Q_{y_i y_i}}$$

$$m_{zi} = \sigma_0 \cdot \sqrt{Q_{z_i z_i}}$$

$$m_{xp1} = 0.4242 \text{ mm}$$

$$m_{yp5} = 0.4188 \text{ mm}$$

$$m_{z1} = 0,6392 \text{ "}$$

$$m_{yp1} = 0.4843 \text{ mm}$$

$$m_{xp6} = 0.5019 \text{ mm}$$

$$m_{z2} = 0.6404 \text{ "}$$

$$m_{xp2} = 0.3303 \text{ mm}$$

$$m_{yp6} = 0.3924 \text{ mm}$$

$$m_{z3} = 0.4920 \text{ "}$$

$$m_{yp2} = 0.5846 \text{ mm}$$

$$m_{xp7} = 0.3273 \text{ mm}$$

$$m_{z4} = 0.4978 \text{ "}$$

$$m_{xp3} = 0.2554 \text{ mm}$$

$$m_{yp7} = 0.4516 \text{ mm}$$

$$m_{z5} = 0.5912 \text{ "}$$

$$m_{yp3} = 0.4235 \text{ mm}$$

$$m_{xp8} = 0.3548 \text{ mm}$$

$$m_{z6} = 0.7795 \text{ "}$$

$$m_{xp4} = 0.3652 \text{ mm}$$

$$m_{yp8} = 0.4285 \text{ mm}$$

$$m_{z7} = 0.6879 \text{ "}$$

$$m_{yp4} = 0.3810 \text{ mm}$$

$$m_{xp9} = 0.6319 \text{ mm}$$

$$m_{z8} = 0.8002 \text{ "}$$

$$m_{xp5} = 0.3243 \text{ mm}$$

$$m_{yp9} = 0.6085 \text{ mm}$$

•Standardno odstupanje položaja tačke

$$m_{pi} = \sqrt{m_{xpi}^2 + m_{ypi}^2}$$

$$m_{p1} = 0.6438 \text{ mm}$$

$$m_{p2} = 0.6715 \text{ mm}$$

$$m_{p3} = 0.4945 \text{ mm}$$

$$m_{p4} = 0.5278 \text{ mm}$$

$$m_{p5} = 0.5297 \text{ mm}$$

$$m_{p6} = 0.6371 \text{ mm}$$

$$m_{p7} = 0.5578 \text{ mm}$$

$$m_{p8} = 0.5563 \text{ mm}$$

$$m_{p9} = 0.8773 \text{ mm}$$

•Elementi elipse grešaka

$$k = \sqrt{(Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2} \quad \lambda_1 = \frac{1}{2}(Q_{xx} + Q_{yy} + k) \quad \lambda_2 = \frac{1}{2}(Q_{xx} + Q_{yy} - k)$$

$$c = \sigma_0 \sqrt{\chi_{1-\alpha,2}^2} \quad A = c\sqrt{\lambda_1} \quad B = c\sqrt{\lambda_2} \quad \operatorname{tg} 2\theta = \frac{2Q_{xy}}{Q_{xx} - Q_{yy}}$$

A – velika poluosa elipse, B – mala poluosa elipse,

θ – ugao koji velika poluosa A zaklapa sa pravcem severa

$A_{P1} = 1.1940 \text{ mm}$	$B_{P1} = 1.0285 \text{ mm}$	$\theta_{P1} = 346^\circ 25' 16''$	$\frac{A_{P1}}{B_{P1}} = 1.1608$
$A_{P2} = 1.4712 \text{ mm}$	$B_{P2} = 0.7328 \text{ mm}$	$\theta_{P2} = 15^\circ 31' 39''$	$\frac{A_{P2}}{B_{P2}} = 2.0076$
$A_{P3} = 1.0619 \text{ mm}$	$B_{P3} = 0.5811 \text{ mm}$	$\theta_{P3} = 15^\circ 00' 50''$	$\frac{A_{P3}}{B_{P3}} = 1.8273$
$A_{P4} = 0.9982 \text{ mm}$	$B_{P4} = 0.8202 \text{ mm}$	$\theta_{P4} = 321^\circ 18' 43''$	$\frac{A_{P4}}{B_{P4}} = 1.2170$
$A_{P5} = 1.0404 \text{ mm}$	$B_{P5} = 0.7738 \text{ mm}$	$\theta_{P5} = 14^\circ 48' 14''$	$\frac{A_{P5}}{B_{P5}} = 1.3446$
$A_{P6} = 1.2516 \text{ mm}$	$B_{P6} = 0.9301 \text{ mm}$	$\theta_{P6} = 16^\circ 36' 27''$	$\frac{A_{P6}}{B_{P6}} = 1.3456$
$A_{P7} = 1.1486 \text{ mm}$	$B_{P7} = 0.7380 \text{ mm}$	$\theta_{P7} = 339^\circ 15' 33''$	$\frac{A_{P7}}{B_{P7}} = 1.5565$
$A_{P8} = 1.0570 \text{ mm}$	$B_{P8} = 0.8585 \text{ mm}$	$\theta_{P8} = 339^\circ 09' 55''$	$\frac{A_{P8}}{B_{P8}} = 1.2312$
$A_{P9} = 1.9692 \text{ mm}$	$B_{P9} = 0.8564 \text{ mm}$	$\theta_{P9} = 07^\circ 59' 05''$	$\frac{A_{P9}}{B_{P9}} = 2.2994$

•Matrica kofaktora popravaka $Q_V \quad Q_V = P^{-1} - A Q_x A^T$

$$Q_{Vdiag} = [\begin{matrix} 0.5315 & 0.5747 & 0.4376 & 0.4312 & 0.5620 & 0.4849 & 0.1956 & 0.5243 & 0.5045 & 0.4891 \\ 0.4275 & 0.5152 & 0.6285 & 0.5202 & 0.3867 & 0.1965 & 0.2198 & 0.4339 & 0.3244 & 0.2498 \\ 0.2498 & 0.2199 & 0.1663 & 0.3199 & 0.3111 & 0.4014 & 0.1393 & 1.0366 & 1.1913 & 1.0217 \\ 1.0217 & 1.1520 & 1.1520 & 1.0366 & 0.7573 & 0.9819 & 0.9173 & 0.6714 & 1.0683 & 0.7090 \\ 0.9596 & 0.9078 & 1.0290 & 0.7234 & 1.0290 & 0.6652 & & & & \end{matrix}]$$

- Koeficijenti unutrašnje pouzdanosti r_i

$$r_i = P_{ii} Q_{v_{ii}} \quad r_i \geq 0.30 \Rightarrow \text{Merenje je pouzdano}$$

$r_1 = 0.5315$	$r_{24} = 0.3199$
$r_2 = 0.5747$	$r_{25} = 0.3111$
$r_3 = 0.4377$	$r_{26} = 0.4014$
$r_4 = 0.4312$	$r_{27} = 0.1393$
$r_5 = 0.5620$	$r_{28} = 0.7168$
$r_6 = 0.4849$	$r_{29} = 0.6487$
$r_7 = 0.1956$	$r_{30} = 0.7068$
$r_8 = 0.5243$	$r_{31} = 0.7068$
$r_9 = 0.5045$	$r_{32} = 0.7520$
$r_{10} = 0.4891$	$r_{33} = 0.7520$
$r_{11} = 0.4275$	$r_{34} = 0.7168$
$r_{12} = 0.5152$	$r_{35} = 0.4707$
$r_{13} = 0.6285$	$r_{36} = 0.6968$
$r_{14} = 0.5202$	$r_{37} = 0.6659$
$r_{15} = 0.3867$	$r_{38} = 0.5436$
$r_{16} = 0.1965$	$r_{39} = 0.7500$
$r_{17} = 0.2198$	$r_{40} = 0.5271$
$r_{18} = 0.4339$	$r_{41} = 0.7216$
$r_{19} = 0.3244$	$r_{42} = 0.6688$
$r_{20} = 0.2498$	$r_{43} = 0.7347$
$r_{21} = 0.2498$	$r_{44} = 0.5286$
$r_{22} = 0.2199$	$r_{45} = 0.7347$
$r_{23} = 0.1663$	$r_{46} = 0.5122$

•Marginalna gruba greška G_i koja se može otkriti *Data-snooping* testom

$$|G_i| = \frac{\sigma_0 \sqrt{\lambda_0}}{P_i \sqrt{Q_{v_{ii}}}} \quad |G_i| < 7\sigma_0 \quad 7\sigma_0 = 4.9497$$

$$\lambda_0 = t_{1-\beta_0} + t_{1-\alpha/2} = 0,842 + 1,96 = 2.802$$

$G_1 = 1.4476$	$G_{17} = 0.9308$	$G_{32} = 3.2650$
$G_2 = 1.5052$	$G_{18} = 1.3079$	$G_{33} = 3.2650$
$G_3 = 1.3136$	$G_{19} = 1.1309$	$G_{34} = 2.9237$
$G_4 = 1.3038$	$G_{20} = 0.9923$	$G_{35} = 2.7798$
$G_5 = 1.4885$	$G_{21} = 0.9923$	$G_{36} = 2.7728$
$G_6 = 1.3826$	$G_{22} = 0.9311$	$G_{37} = 2.6196$
$G_7 = 0.8782$	$G_{23} = 0.8096$	$G_{38} = 2.0092$
$G_8 = 1.4377$	$G_{24} = 1.1230$	$G_{39} = 2.9232$
$G_9 = 1.4103$	$G_{25} = 1.1075$	$G_{40} = 2.2489$
$G_{10} = 1.3886$	$G_{26} = 1.2580$	$G_{41} = 2.5866$
$G_{11} = 1.2982$	$G_{27} = 0.7411$	$G_{42} = 2.5677$
$G_{12} = 1.4252$	$G_{28} = 2.9237$	$G_{43} = 2.8210$
$G_{13} = 1.5741$	$G_{29} = 3.9798$	$G_{44} = 2.3112$
$G_{14} = 1.4321$	$G_{30} = 2.9012$	$G_{45} = 2.8210$
$G_{15} = 1.2347$	$G_{31} = 2.9012$	$G_{46} = 2.1033$
$G_{16} = 0.8801$		

ZADATAK 2

a)

•Plan opažanja

Plan opažanja		
od	do	S[km]
P2	P1	0,283
P3	P2	0,308
P3	P1	0,296
P1	P4	0,269
P4	P3	0,281
P5	P4	0,45
P6	P5	0,291
P3	P6	0,454
P5	P8	0,32
P6	P8	0,362
P7	P6	0,35
P7	P8	0,354

•Funkcije veze

$$\Delta h_{p2-p1} = H_{P1} - H_{P2}$$

$$\Delta h_{p6-p5} = H_{P5} - H_{P6}$$

$$\Delta h_{p3-p2} = H_{P3} - H_{P2}$$

$$\Delta h_{p3-p6} = H_{P6} - H_{P3}$$

$$\Delta h_{p3-p1} = H_{P1} - H_{P3}$$

$$\Delta h_{p5-p8} = H_{P8} - H_{P5}$$

$$\Delta h_{p1-p4} = H_{P4} - H_{P1}$$

$$\Delta h_{p6-p8} = H_{P8} - H_{P6}$$

$$\Delta h_{p4-p3} = H_{P3} - H_{P4}$$

$$\Delta h_{p7-p6} = H_{P6} - H_{P7}$$

$$\Delta h_{p5-p4} = H_{P4} - H_{P5}$$

$$\Delta h_{p7-p8} = H_{P8} - H_{P7}$$

•Jednačine popravaka

$$v_{\Delta h_{p2-p1}} = \Delta H_{P1} - \Delta H_{P2} + f_{\Delta h_{p2-p1}}$$

$$v_{\Delta h_{p6-p5}} = \Delta H_{P5} - \Delta H_{P6} + f_{\Delta h_{p6-p5}}$$

$$v_{\Delta h_{p3-p2}} = \Delta H_{P2} - \Delta H_{P3} + f_{\Delta h_{p3-p2}}$$

$$v_{\Delta h_{p3-p6}} = \Delta H_{P6} - \Delta H_{P3} + f_{\Delta h_{p3-p6}}$$

$$v_{\Delta h_{p3-p1}} = \Delta H_{P1} - \Delta H_{P3} + f_{\Delta h_{p3-p1}}$$

$$v_{\Delta h_{p5-p8}} = \Delta H_{P8} - \Delta H_{P5} + f_{\Delta h_{p5-p8}}$$

$$v_{\Delta h_{p1-p4}} = \Delta H_{P4} - \Delta H_{P1} + f_{\Delta h_{p1-p4}}$$

$$v_{\Delta h_{p6-p8}} = \Delta H_{P8} - \Delta H_{P6} + f_{\Delta h_{p6-p8}}$$

$$v_{\Delta h_{p4-p3}} = \Delta H_{P3} - \Delta H_{P4} + f_{\Delta h_{p4-p3}}$$

$$v_{\Delta h_{p7-p6}} = \Delta H_{P6} - \Delta H_{P7} + f_{\Delta h_{p7-p6}}$$

$$v_{\Delta h_{p5-p4}} = \Delta H_{P4} - \Delta H_{P5} + f_{\Delta h_{p5-p4}}$$

$$v_{\Delta h_{p7-p8}} = \Delta H_{P8} - \Delta H_{P7} + f_{\Delta h_{p7-p8}}$$

•Matrica dizajna A

$$A = \begin{array}{c|cccccccc} & \text{H1} & \text{H2} & \text{H3} & \text{H4} & \text{H5} & \text{H6} & \text{H7} & \text{H8} \\ \hline & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}$$

•Matrica kofaktora nepoznatih parametara $N^- = Q_x \begin{bmatrix} N^- & R^- \\ R^{T-} & 0^- \end{bmatrix}$

$$\begin{bmatrix} N^- & R^- \\ R^{T-} & 0^- \end{bmatrix} = \begin{array}{cccccccc|c} 0 & -9E-17 & -9E-17 & -3E-17 & -0 & -1,9E-17 & -5E-17 & -1E-16 & 1 \\ 0 & 0,1623 & 0,0609 & 0,0332 & 0,045 & 0,049413 & 0,04839 & 0,047 & 1 \\ 0 & 0,0609 & 0,1271 & 0,0694 & 0,093 & 0,103192 & 0,10105 & 0,099 & 1 \\ 0 & 0,0332 & 0,0694 & 0,1481 & 0,116 & 0,102025 & 0,10495 & 0,108 & 1 \\ 0 & 0,0446 & 0,0931 & 0,1158 & 0,346 & 0,270829 & 0,28676 & 0,303 & 1 \\ 0 & 0,0494 & 0,1032 & 0,102 & 0,271 & 0,342622 & 0,32736 & 0,312 & 1 \\ 0 & 0,0484 & 0,101 & 0,1049 & 0,287 & 0,327359 & 0,50809 & 0,371 & 1 \\ 0 & 0,0473 & 0,0989 & 0,1079 & 0,303 & 0,311922 & 0,37139 & 0,432 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -0 \end{array}$$

•Standardno odstupanje nepozatih parametara

$$m_{Hi} = \sigma_0 \cdot \sqrt{Q_{x_{ii}}}$$

$$m_{H1} = 0.0000 \text{ mm}$$

$$m_{H5} = 0.5880 \text{ mm}$$

$$m_{H2} = 0.4028 \text{ mm}$$

$$m_{H6} = 0.5853 \text{ mm}$$

$$m_{H3} = 0.3565 \text{ mm}$$

$$m_{H7} = 0.7128 \text{ mm}$$

$$m_{H4} = 0.3848 \text{ mm}$$

$$m_{H8} = 0.6569 \text{ mm}$$

•Matrica kofaktora popravaka $Q_V = Q_v = P^{-1} - A Q_x A^T$

$$Q_v = \begin{array}{cccccccccccc|c} 0,0932 & 0,1014 & -0,061 & 0,0332 & 0,028 & -0,0114 & -0,0048 & -0,012 & 0,003 & -0,002 & 0,001 & -0,001 \\ 0,1014 & 0,1103 & -0,066 & 0,0362 & 0,03 & -0,0124 & -0,0053 & -0,013 & 0,003 & -0,002 & 0,0011 & -0,0011 \\ -0,0609 & -0,066 & 0,14 & 0,0694 & 0,058 & -0,0237 & -0,0101 & -0,024 & 0,006 & -0,004 & 0,0021 & -0,0022 \\ 0,0332 & 0,0362 & 0,0694 & 0,0947 & 0,079 & -0,0323 & -0,0138 & -0,033 & 0,008 & -0,006 & 0,0029 & -0,003 \\ 0,0276 & 0,0301 & 0,0577 & 0,0787 & 0,117 & 0,056 & 0,0238 & 0,057 & -0,01 & 0,0102 & -0,005 & 0,0051 \\ -0,0114 & -0,012 & -0,024 & -0,032 & 0,056 & 0,1438 & 0,0612 & 0,145 & -0,04 & 0,0262 & -0,013 & 0,0132 \\ -0,0048 & -0,005 & -0,01 & -0,014 & 0,024 & 0,0612 & 0,1159 & 0,062 & 0,084 & -0,063 & 0,0312 & -0,0315 \\ -0,0115 & -0,013 & -0,024 & -0,033 & 0,057 & 0,1451 & 0,0617 & 0,146 & -0,04 & 0,0264 & -0,013 & 0,0133 \\ 0,0028 & 0,003 & 0,0058 & 0,0079 & -0,01 & -0,035 & 0,084 & -0,035 & 0,117 & -0,088 & 0,0435 & -0,044 \\ -0,0021 & -0,002 & -0,004 & -0,006 & 0,01 & 0,0262 & -0,0627 & 0,026 & -0,09 & 0,1764 & 0,0747 & -0,0756 \\ 0,001 & 0,0011 & 0,0021 & 0,0029 & -0,01 & -0,013 & 0,0312 & -0,013 & 0,044 & 0,0747 & 0,1199 & -0,1213 \\ -0,001 & -0,001 & -0,002 & -0,003 & 0,005 & 0,0132 & -0,0315 & 0,013 & -0,04 & -0,076 & -0,121 & 0,1226 \end{array}$$

- Koeficijenti unutrašnje pouzdanosti r_i

$$r_i = P_{ii} Q_{v_{ii}} \quad r_i \geq 0.30 \Rightarrow \text{Merenje je pouzdano}$$

$r_1 = 0.3649$	$r_7 = 0.4413$
$r_2 = 0.3968$	$r_8 = 0.3573$
$r_3 = 0.5241$	$r_9 = 0.4058$
$r_4 = 0.3901$	$r_{10} = 0.5399$
$r_5 = 0.4621$	$r_{11} = 0.3796$
$r_6 = 0.3541$	$r_{12} = 0.3837$

- Marginalna gruba greška G_i koja se može otkriti *Data-snooping* testom

$$|G_i| = \frac{\sigma_0 \sqrt{\lambda_0}}{P_i \sqrt{Q_{v_{ii}}}} \quad \sigma_0 = 1$$

$$\lambda_0 = t_{1-\beta_0} + t_{1-\alpha/2} = 0,842 + 1,96 = 2,802$$

$G_1 = 2.3442$	$G_7 = 2.1616$
$G_2 = 2.3452$	$G_8 = 3.0005$
$G_3 = 2.0005$	$G_9 = 2.3637$
$G_4 = 2.2105$	$G_{10} = 2.1796$
$G_5 = 2.0757$	$G_{11} = 2.5561$
$G_6 = 3.0009$	$G_{12} = 2.5567$

b)

•Plan opažanja

Plan opažanja		
od	do	S[km]
P2	P1	0,283
P3	P2	0,308
P3	P1	0,296
P1	P4	0,269
P4	P3	0,281
P5	P4	0,45
P6	P5	0,291
P3	P6	0,454
P5	P8	0,32
P6	P8	0,362
P7	P6	0,35
P7	P8	0,354

•Funkcije veze

$$\Delta h_{p2-p1} = H_{P1} - H_{P2}$$

$$\Delta h_{p6-p5} = H_{P5} - H_{P6}$$

$$\Delta h_{p3-p2} = H_{P3} - H_{P2}$$

$$\Delta h_{p3-p6} = H_{P6} - H_{P3}$$

$$\Delta h_{p3-p1} = H_{P1} - H_{P3}$$

$$\Delta h_{p5-p8} = H_{P8} - H_{P5}$$

$$\Delta h_{p1-p4} = H_{P4} - H_{P1}$$

$$\Delta h_{p6-p8} = H_{P8} - H_{P6}$$

$$\Delta h_{p4-p3} = H_{P3} - H_{P4}$$

$$\Delta h_{p7-p6} = H_{P6} - H_{P7}$$

$$\Delta h_{p5-p4} = H_{P4} - H_{P5}$$

$$\Delta h_{p7-p8} = H_{P8} - H_{P7}$$

•Jednačine popravaka

$$v_{\Delta h_{p2-p1}} = \Delta H_{P1} - \Delta H_{P2} + f_{\Delta h_{p2-p1}}$$

$$v_{\Delta h_{p6-p5}} = \Delta H_{P5} - \Delta H_{P6} + f_{\Delta h_{p6-p5}}$$

$$v_{\Delta h_{p3-p2}} = \Delta H_{P2} - \Delta H_{P3} + f_{\Delta h_{p3-p2}}$$

$$v_{\Delta h_{p3-p6}} = \Delta H_{P6} - \Delta H_{P3} + f_{\Delta h_{p3-p6}}$$

$$v_{\Delta h_{p3-p1}} = \Delta H_{P1} - \Delta H_{P3} + f_{\Delta h_{p3-p1}}$$

$$v_{\Delta h_{p5-p8}} = \Delta H_{P8} - \Delta H_{P5} + f_{\Delta h_{p5-p8}}$$

$$v_{\Delta h_{p1-p4}} = \Delta H_{P4} - \Delta H_{P1} + f_{\Delta h_{p1-p4}}$$

$$v_{\Delta h_{p6-p8}} = \Delta H_{P8} - \Delta H_{P6} + f_{\Delta h_{p6-p8}}$$

$$v_{\Delta h_{p4-p3}} = \Delta H_{P3} - \Delta H_{P4} + f_{\Delta h_{p4-p3}}$$

$$v_{\Delta h_{p7-p6}} = \Delta H_{P6} - \Delta H_{P7} + f_{\Delta h_{p7-p6}}$$

$$v_{\Delta h_{p5-p4}} = \Delta H_{P4} - \Delta H_{P5} + f_{\Delta h_{p5-p4}}$$

$$v_{\Delta h_{p7-p8}} = \Delta H_{P8} - \Delta H_{P7} + f_{\Delta h_{p7-p8}}$$

•Matrica dizajna A

$$A = \begin{array}{c|cccccccc} & \text{H1} & \text{H2} & \text{H3} & \text{H4} & \text{H5} & \text{H6} & \text{H7} & \text{H8} \\ \hline & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}$$

•Matrica kofaktora nepoznatih parametara $N^+ = Q_x \begin{bmatrix} N^+ & B^+ \\ B^{T+} & 0^+ \end{bmatrix}$

$$\begin{bmatrix} N^+ & B^+ \\ B^{T+} & 0^+ \end{bmatrix} = \begin{bmatrix} 0,1276 & 0,0719 & 0,0459 & 0,0424 & -0,05 & -0,0608 & -0,0909 & -0,081 & 0,125 \\ 0,0719 & 0,1784 & 0,051 & 0,0199 & -0,07 & -0,06714 & -0,0982 & -0,09 & 0,125 \\ 0,0459 & 0,051 & 0,0913 & 0,0302 & -0,04 & -0,03931 & -0,0715 & -0,064 & 0,125 \\ 0,0424 & 0,0199 & 0,0302 & 0,1054 & -0,02 & -0,04395 & -0,0711 & -0,059 & 0,125 \\ -0,0548 & -0,066 & -0,043 & -0,024 & 0,108 & 0,027575 & 0,01342 & 0,039 & 0,125 \\ -0,0608 & -0,067 & -0,039 & -0,044 & 0,028 & 0,093406 & 0,04807 & 0,042 & 0,125 \\ -0,0909 & -0,098 & -0,072 & -0,071 & 0,013 & 0,048066 & 0,19872 & 0,072 & 0,125 \\ -0,0814 & -0,09 & -0,064 & -0,059 & 0,039 & 0,042143 & 0,07154 & 0,141 & 0,125 \\ \hline 0,125 & 0,125 & 0,125 & 0,125 & 0,125 & 0,125 & 0,125 & 0,125 & -0 \end{bmatrix}$$

•Standardno odstupanje nepozatih parametara

$$m_{Hi} = \sigma_0 \cdot \sqrt{Q_{xii}}$$

$$m_{H1} = 0.3572 \text{ mm}$$

$$m_{H5} = 0.3293 \text{ mm}$$

$$m_{H2} = 0.4223 \text{ mm}$$

$$m_{H6} = 0.3056 \text{ mm}$$

$$m_{H3} = 0.3022 \text{ mm}$$

$$m_{H7} = 0.4458 \text{ mm}$$

$$m_{H4} = 0.3246 \text{ mm}$$

$$m_{H8} = 0.3758 \text{ mm}$$

•Matrica kofaktora popravaka $Q_V \quad Q_V = P^{-1} - A Q_x A^T$

$$Q_V = \begin{bmatrix} 0,0932 & 0,1014 & -0,061 & 0,0332 & 0,028 & -0,01135 & -0,0048 & -0,011 & 0,003 & -0,002 & 0,001 & -0,001 \\ 0,1014 & 0,1103 & -0,066 & 0,0362 & 0,03 & -0,01235 & -0,0053 & -0,012 & 0,003 & -0,002 & 0,0011 & -0,001 \\ -0,0609 & -0,066 & 0,14 & 0,0694 & 0,058 & -0,0237 & -0,0101 & -0,024 & 0,006 & -0,004 & 0,0021 & -0,002 \\ 0,0332 & 0,0362 & 0,0694 & 0,0947 & 0,079 & -0,03233 & -0,0138 & -0,033 & 0,008 & -0,006 & 0,0029 & -0,003 \\ 0,0276 & 0,0301 & 0,0577 & 0,0787 & 0,117 & 0,056035 & 0,02383 & 0,057 & -0,01 & 0,0102 & -0,005 & 0,0051 \\ -0,0114 & -0,012 & -0,024 & -0,032 & 0,056 & 0,143822 & 0,06117 & 0,145 & -0,04 & 0,0262 & -0,013 & 0,0132 \\ -0,0048 & -0,005 & -0,01 & -0,014 & 0,024 & 0,061167 & 0,11592 & 0,062 & 0,084 & -0,063 & 0,0312 & -0,032 \\ -0,0115 & -0,012 & -0,024 & -0,033 & 0,057 & 0,1451 & 0,06171 & 0,146 & -0,04 & 0,0264 & -0,013 & 0,0133 \\ 0,0028 & 0,003 & 0,0058 & 0,0079 & -0,01 & -0,03501 & 0,08397 & -0,035 & 0,117 & -0,088 & 0,0435 & -0,044 \\ -0,0021 & -0,002 & -0,004 & -0,006 & 0,01 & 0,026156 & -0,0627 & 0,026 & -0,09 & 0,1764 & 0,0747 & -0,076 \\ 0,001 & 0,0011 & 0,0021 & 0,0029 & -0,01 & -0,013 & 0,03119 & -0,013 & 0,044 & 0,0747 & 0,1199 & -0,121 \\ -0,001 & -0,001 & -0,002 & -0,003 & 0,005 & 0,013152 & -0,0315 & 0,013 & -0,04 & -0,076 & -0,121 & 0,1226 \end{bmatrix}$$

- Koeficijenti unutrašnje pouzdanosti r_i

$$r_i = P_{ii} Q_{v_{ii}} \quad r_i \geq 0.30 \Rightarrow \text{Merenje je pouzdano}$$

$$r_1 = 0.3647 \quad r_7 = 0.4414$$

$$r_2 = 0.3970 \quad r_8 = 0.3573$$

$$r_3 = 0.5242 \quad r_9 = 0.4060$$

$$r_4 = 0.3899 \quad r_{10} = 0.5399$$

$$r_5 = 0.4622 \quad r_{11} = 0.3795$$

$$r_6 = 0.3541 \quad r_{12} = 0.3839$$

- Marginalna gruba greška G_i koja se može otkriti *Data-snooping* testom

$$|G_i| = \frac{\sigma_0 \sqrt{\lambda_0}}{P_i \sqrt{Q_{v_{ii}}}} \quad \sigma_0 = 1$$

$$\lambda_0 = t_{1-\beta_0} + t_{1-\alpha/2} = 0,842 + 1,96 = 2,802$$

$$G_1 = 2.3447 \quad G_7 = 2.1614$$

$$G_2 = 2.3447 \quad G_8 = 3.0006$$

$$G_3 = 2.0003 \quad G_9 = 2.3634$$

$$G_4 = 2.2109 \quad G_{10} = 2.1797$$

$$G_5 = 2.0757 \quad G_{11} = 2.5562$$

$$G_6 = 3.0006 \quad G_{12} = 2.5562$$

ZADATAK 3

• Matrica težina P

$$A = E \quad E - \text{jedinična matrica}$$

$$N = A^T P A \quad N = P$$

$$P_{xi} = \frac{\sigma_0}{\sigma_{xi}} \quad P_{yi} = \frac{\sigma_0}{\sigma_{yi}} \quad \sigma_0 = 1$$

$$P = \begin{vmatrix} 0,147929 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0,127551 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,147929 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,127551 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,147929 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0,127551 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0,147929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,127551 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,127551 \end{vmatrix}$$

• Matrica kofaktora nepoznatih parametara $Q_x \quad Q_x = N^{-1}$

$$Q_x = \begin{vmatrix} 6,76 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7,84 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6,76 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7,84 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6,76 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7,84 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6,76 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6,76 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7,84 \end{vmatrix}$$

• Postavljanje hipoteza

H_0 : Obeležena figura je kvadrat.

H_a : Obeležena figura nije kvadrat.

$$H_0: M \begin{bmatrix} d_{P1-P2} - d_{P2-P3} \\ d_{P2-P3} - d_{P3-P4} \\ d_{P3-P4} - d_{P4-P1} \\ v_{P1}^{P4} - v_{P1}^{P2} - 90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_a: M \begin{bmatrix} d_{P1-P2} - d_{P2-P3} \\ d_{P2-P3} - d_{P3-P4} \\ d_{P3-P4} - d_{P4-P1} \\ v_{P1}^{P4} - v_{P1}^{P2} - 90^\circ \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

•Funkcije veze

$$L_1: d_{P1-P2} - d_{P2-P3} = \sqrt{[(Y_{P2} - Y_{P1})^2 + (X_{P2} - X_{P1})^2]} - \sqrt{[(Y_{P3} - Y_{P2})^2 + (X_{P3} - X_{P2})^2]}$$

$$L_2: d_{P2-P3} - d_{P3-P4} = \sqrt{[(Y_{P3} - Y_{P2})^2 + (X_{P3} - X_{P2})^2]} - \sqrt{[(Y_{P4} - Y_{P3})^2 + (X_{P4} - X_{P3})^2]}$$

$$L_3: d_{P3-P4} - d_{P4-P1} = \sqrt{[(Y_{P4} - Y_{P3})^2 + (X_{P4} - X_{P3})^2]} - \sqrt{[(Y_{P1} - Y_{P4})^2 + (X_{P1} - X_{P4})^2]}$$

$$L_4: \nu_{P1}^{P4} - \nu_{P1}^{P2} - 90^\circ = \tan^{-1} \frac{Y_{P4} - Y_{P1}}{X_{P4} - X_{P1}} - \tan^{-1} \frac{Y_{P2} - Y_{P1}}{X_{P2} - X_{P1}} - 90^\circ$$

•Matrica koeficijenata H

$$H = \begin{vmatrix} \frac{\partial L_1}{\partial X_{P1}} & \frac{\partial L_1}{\partial Y_{P1}} & \frac{\partial L_1}{\partial X_{P2}} & \frac{\partial L_1}{\partial Y_{P2}} & \frac{\partial L_1}{\partial X_{P3}} & \frac{\partial L_1}{\partial Y_{P3}} & \frac{\partial L_1}{\partial X_{P4}} & \frac{\partial L_1}{\partial Y_{P4}} \\ \frac{\partial L_2}{\partial X_{P1}} & \frac{\partial L_2}{\partial Y_{P1}} & \frac{\partial L_2}{\partial X_{P2}} & \frac{\partial L_2}{\partial Y_{P2}} & \frac{\partial L_2}{\partial X_{P3}} & \frac{\partial L_2}{\partial Y_{P3}} & \frac{\partial L_2}{\partial X_{P4}} & \frac{\partial L_2}{\partial Y_{P4}} \\ \frac{\partial L_3}{\partial X_{P1}} & \frac{\partial L_3}{\partial Y_{P1}} & \frac{\partial L_3}{\partial X_{P2}} & \frac{\partial L_3}{\partial Y_{P2}} & \frac{\partial L_3}{\partial X_{P3}} & \frac{\partial L_3}{\partial Y_{P3}} & \frac{\partial L_3}{\partial X_{P4}} & \frac{\partial L_3}{\partial Y_{P4}} \\ \frac{\partial L_4}{\partial X_{P1}} & \frac{\partial L_4}{\partial Y_{P1}} & \frac{\partial L_4}{\partial X_{P2}} & \frac{\partial L_4}{\partial Y_{P2}} & \frac{\partial L_4}{\partial X_{P3}} & \frac{\partial L_4}{\partial Y_{P3}} & \frac{\partial L_4}{\partial X_{P4}} & \frac{\partial L_4}{\partial Y_{P4}} \end{vmatrix}$$

$$H = \begin{vmatrix} \text{XP1} & \text{YP1} & \text{XP2} & \text{YP2} & \text{XP3} & \text{YP3} & \text{XP4} & \text{YP4} \\ -0,89372 & -0,44862 & 0,445029 & 1,3423 & 0,448695 & -0,89368 & 0 & 0 \\ 0 & 0 & 0,448695 & -0,89368 & -1,34246 & 0,445149 & 0,893765 & 0,4485353 \\ -0,44864 & 0,893713 & 0 & 0 & 0,893765 & 0,448535 & -0,44513 & -1,342249 \\ 0,766298 & 2,31108 & 0,772359 & -1,53868 & 0 & 0 & -1,53866 & -0,772397 \end{vmatrix}$$

•Vektor odstupanja d

$$d = \begin{bmatrix} d_{P1-P2} - d_{P2-P3} \\ d_{P2-P3} - d_{P3-P4} \\ d_{P3-P4} - d_{P4-P1} \\ \nu_{P1}^{P4} - \nu_{P1}^{P2} - 90^\circ \end{bmatrix} \quad d = \begin{bmatrix} 10,26859 \\ -2,64962 \\ -8,07113 \\ 5,388385 \end{bmatrix}$$

•Matrica Q_d $Q_d = HQ_xH^T$

$$Q_d = \begin{vmatrix} 30,06462 & -15,2458 & -0,86454 & -26,627 \\ -15,245814 & 28,33628 & -13,955 & 1,110987 \\ -0,8645371 & -13,955 & 30,0641 & 26,62706 \\ -26,627026 & 1,110987 & 26,62706 & 89,11928 \end{vmatrix}$$

•Matrica Q_d^{-1}

$$Q_d^{-1} = \begin{vmatrix} 0,070209 & 0,037997 & 0,002036 & 0,019895 \\ 0,037997 & 0,072948 & 0,034953 & 8,15E-08 \\ 0,002036 & 0,034953 & 0,067166 & -0,0199 \\ 0,019895 & 8,15E-08 & -0,0199 & 0,02311 \end{vmatrix}$$

•Test statistika

$$T = \frac{d^T Q_d^{-1} d}{k\sigma_0^2} \approx F_{0,95}(k, \infty)$$

k – rang matrice H , $F_{0,95}(k, \infty)$ – kvantil Fišerovog rasporeda

$$k = \text{rang}(H) = 4$$

$$T = 3.9959 > F_{0,95}(4, \infty) = 2.3728$$

ZAKLJUČAK:

Nulta hipoteza H_0 se odbacuje. Prihvata se alternativna hipoteza H_a : obeležena figura nije kvadrat.

ZADATAK 4

- Matrica kofaktora nepoznatih parametara Q_x

$$P_i = \frac{\sigma_0^2}{\sigma_i^2} \quad \sigma_0 = 6.8 \text{ mm} \quad \sigma_i = 6.8 \text{ mm}$$

$$P = N = Q_x$$

$$Q_x = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

- Postavljanje hipoteza

H_0 : Tačke pripadaju istoj pravoj.

H_a : Tačke ne pripadaju istoj pravoj.

$$H_0: M \begin{bmatrix} v_1^2 - v_2^3 \\ v_2^3 - v_3^4 \\ v_3^4 - v_4^5 \\ v_4^5 - v_5^6 \\ v_5^6 - v_6^7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_a: M \begin{bmatrix} v_1^2 - v_2^3 \\ v_2^3 - v_3^4 \\ v_3^4 - v_4^5 \\ v_4^5 - v_5^6 \\ v_5^6 - v_6^7 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

•Funkcije veze

$$L_1: v_1^2 - v_2^3 = \tan^{-1} \frac{Y_2 - Y_1}{X_2 - X_1} - \tan^{-1} \frac{Y_3 - Y_2}{X_3 - X_2}$$

$$L_2: v_2^3 - v_3^4 = \tan^{-1} \frac{Y_3 - Y_2}{X_3 - X_2} - \tan^{-1} \frac{Y_4 - Y_3}{X_4 - X_3}$$

$$L_3: v_3^4 - v_4^5 = \tan^{-1} \frac{Y_4 - Y_3}{X_4 - X_3} - \tan^{-1} \frac{Y_5 - Y_4}{X_5 - X_4}$$

$$L_4: v_4^5 - v_5^6 = \tan^{-1} \frac{Y_5 - Y_4}{X_5 - X_4} - \tan^{-1} \frac{Y_6 - Y_5}{X_6 - X_5}$$

$$L_5: v_5^6 - v_6^7 = \tan^{-1} \frac{Y_6 - Y_5}{X_6 - X_5} - \tan^{-1} \frac{Y_7 - Y_6}{X_7 - X_6}$$

•Matrica koeficijenata H

$$H = \begin{pmatrix} \frac{\partial L_1}{\partial X_1} & \frac{\partial L_1}{\partial Y_1} & \frac{\partial L_1}{\partial X_2} & \frac{\partial L_1}{\partial Y_2} & \frac{\partial L_1}{\partial X_3} & \frac{\partial L_1}{\partial Y_3} & \frac{\partial L_1}{\partial X_4} & \frac{\partial L_1}{\partial Y_4} & \frac{\partial L_1}{\partial X_5} & \frac{\partial L_1}{\partial Y_5} & \frac{\partial L_1}{\partial X_6} & \frac{\partial L_1}{\partial Y_6} & \frac{\partial L_1}{\partial X_7} & \frac{\partial L_1}{\partial Y_7} \\ \frac{\partial L_2}{\partial X_1} & \frac{\partial L_2}{\partial Y_1} & \frac{\partial L_2}{\partial X_2} & \frac{\partial L_2}{\partial Y_2} & \frac{\partial L_2}{\partial X_3} & \frac{\partial L_2}{\partial Y_3} & \frac{\partial L_2}{\partial X_4} & \frac{\partial L_2}{\partial Y_4} & \frac{\partial L_2}{\partial X_5} & \frac{\partial L_2}{\partial Y_5} & \frac{\partial L_2}{\partial X_6} & \frac{\partial L_2}{\partial Y_6} & \frac{\partial L_2}{\partial X_7} & \frac{\partial L_2}{\partial Y_7} \\ \frac{\partial L_3}{\partial X_1} & \frac{\partial L_3}{\partial Y_1} & \frac{\partial L_3}{\partial X_2} & \frac{\partial L_3}{\partial Y_2} & \frac{\partial L_3}{\partial X_3} & \frac{\partial L_3}{\partial Y_3} & \frac{\partial L_3}{\partial X_4} & \frac{\partial L_3}{\partial Y_4} & \frac{\partial L_3}{\partial X_5} & \frac{\partial L_3}{\partial Y_5} & \frac{\partial L_3}{\partial X_6} & \frac{\partial L_3}{\partial Y_6} & \frac{\partial L_3}{\partial X_7} & \frac{\partial L_3}{\partial Y_7} \\ \frac{\partial L_4}{\partial X_1} & \frac{\partial L_4}{\partial Y_1} & \frac{\partial L_4}{\partial X_2} & \frac{\partial L_4}{\partial Y_2} & \frac{\partial L_4}{\partial X_3} & \frac{\partial L_4}{\partial Y_3} & \frac{\partial L_4}{\partial X_4} & \frac{\partial L_4}{\partial Y_4} & \frac{\partial L_4}{\partial X_5} & \frac{\partial L_4}{\partial Y_5} & \frac{\partial L_4}{\partial X_6} & \frac{\partial L_4}{\partial Y_6} & \frac{\partial L_4}{\partial X_7} & \frac{\partial L_4}{\partial Y_7} \\ \frac{\partial L_5}{\partial X_1} & \frac{\partial L_5}{\partial Y_1} & \frac{\partial L_5}{\partial X_2} & \frac{\partial L_5}{\partial Y_2} & \frac{\partial L_5}{\partial X_3} & \frac{\partial L_5}{\partial Y_3} & \frac{\partial L_5}{\partial X_4} & \frac{\partial L_5}{\partial Y_4} & \frac{\partial L_5}{\partial X_5} & \frac{\partial L_5}{\partial Y_5} & \frac{\partial L_5}{\partial X_6} & \frac{\partial L_5}{\partial Y_6} & \frac{\partial L_5}{\partial X_7} & \frac{\partial L_5}{\partial Y_7} \end{pmatrix}$$

$$H = \begin{pmatrix} X1 & Y1 & X2 & Y2 & X3 & Y3 & X4 & Y4 & X5 & Y5 & X6 & Y6 & X7 & Y7 \\ 20,6265 & -0,0206 & -41,253 & 0,0619 & 20,6264 & -0,0413 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20,6264 & -0,041 & -41,253 & 0,0825 & 20,6264 & -0,0413 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20,6264 & -0,0413 & -41,253 & 0,0619 & 20,6265 & -0,0206 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20,6265 & -0,0206 & -41,253 & 0,0413 & 20,6265 & -0,0206 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20,6265 & -0,0206 & -41,252 & -0,1031 & 20,6258 & 0,1238 \end{pmatrix}$$

•Vektor odstupanja d

$$d = \begin{bmatrix} v_1^2 - v_2^3 \\ v_2^3 - v_3^4 \\ v_3^4 - v_4^5 \\ v_4^5 - v_5^6 \\ v_5^6 - v_6^7 \end{bmatrix} \quad d = \begin{bmatrix} 206,26432 \\ -1,023E-10 \\ -206,26432 \\ 0 \\ -1443,8387 \end{bmatrix}$$

•Matrica Q_d $Q_d = H Q_x H^T$

$$Q_d = \begin{bmatrix} 2552,71 & -1701,8 & 425,451 & 0 & 0 \\ -1701,8 & 2552,7 & -1701,8 & 425,45 & 0 \\ 425,451 & -1701,8 & 2552,71 & -1702 & 425,452 \\ 0 & 425,45 & -1701,8 & 2552,7 & -1701,8 \\ 0 & 0 & 425,452 & -1702 & 2552,65 \end{bmatrix}$$

•Matrica Q_d^{-1}

$$Q_d^{-1} = \begin{bmatrix} 0,0013 & 0,00168 & 0,0015 & 0,00101 & 0,0004 \\ 0,0017 & 0,00336 & 0,0034 & 0,00235 & 0,001 \\ 0,0015 & 0,00336 & 0,0044 & 0,00336 & 0,0015 \\ 0,001 & 0,00235 & 0,0034 & 0,00336 & 0,0017 \\ 0,0004 & 0,00101 & 0,0015 & 0,00168 & 0,0013 \end{bmatrix}$$

•Test statistika

$$T = \frac{d^T Q_d^{-1} d}{k \sigma_0^2} \approx F_{0,95}(k, \infty)$$

k – rang matrice H , $F_{0,95}(k, \infty)$ – kvantil Fišerovog rasporeda

$$k = \text{rang}(H) = 5$$

$$T = 14.6441 > F_{0,95}(5, \infty) = 2.2150$$

ZAKLJUČAK:

Nulta hipoteza H_0 se odbacuje. Prihvata se alternativna hipoteza H_a : tačke ne pripadaju istoj pravoj.

ZADATAK 5

$$\alpha = X \quad \alpha_0 = X_0 \quad \gamma_0 = 180 - (X_0 + Y_0)$$

$$\beta = Y \quad \beta_0 = Y_0$$

•Jednačine popravaka

$$\alpha + v_\alpha = \alpha \quad v_\alpha = X - \alpha \quad f_\alpha = 0$$

$$\beta + v_\beta = \beta \quad v_\beta = Y - \beta \quad f_\beta = 0$$

$$\gamma + v_\gamma = 180^\circ - (X + Y) \quad v_\gamma = 180^\circ - (X + Y) \quad f_\gamma = -12''$$

•Matrica dizajna A i matrica težina P

$$A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{vmatrix} \quad P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

•Vektor slobodnih članova f

$$f = \begin{vmatrix} 0 \\ 0 \\ -12 \end{vmatrix}$$

•Matrica koeficijenta normalnih jednacina N i vektor slobodnih članova normalnih jednačina n

$$N = A^T \cdot P \cdot A \quad n = A^T \cdot P \cdot f$$

$$N = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \quad n = \begin{vmatrix} 12 \\ 12 \end{vmatrix}$$

•Vektor nepoznatih parametara \hat{x} i matrica kofaktora $Q_{\hat{x}}$

$$\hat{x} = -N^{-1} \cdot n = -Q_{\hat{x}} \cdot n$$

$$Q_x = \begin{vmatrix} 0,666667 & -0,333333 \\ -0,333333 & 0,666667 \end{vmatrix} \quad \hat{x} = \begin{vmatrix} -4 \\ -4 \end{vmatrix}$$

- Vektor popravaka merenih veličina \hat{V}

$$\hat{V} = A \cdot \hat{x} + f$$

$$V = \begin{vmatrix} -4 \\ -4 \\ -4 \end{vmatrix}$$

- Kontrola izravnjanja

$$\hat{V}^T \cdot P \cdot V = 48 \quad f^T \cdot P \cdot f + n^T \cdot \hat{x} = 48$$

- Aposteriori standardna devijacija

$$m_0 = \sqrt{\frac{\hat{V}^T \cdot P \cdot V}{f}} \quad , f = n - u \quad m_0 = \sqrt{\frac{48}{3-2}} = 6.928203$$

- Globalni test na grube greške

-Hipoteze

$$H_0: \sigma^2 = \sigma_0^2 \quad , \quad H_a: \sigma^2 \neq \sigma_0^2 \quad , \quad \sigma = m_0$$

-Test statistika

$$F = \frac{m_0^2}{\sigma_0^2} = 1.58678 > F(0.05, 1, \infty) = 3.84146$$

=> Prihavata se H_0

Nema grubih grešaka!!

- Izravnate vrednosti

$$\hat{\alpha} = 32^\circ 45' 38''$$

$$\hat{\beta} = 90^\circ 00' 08''$$

$$\hat{\gamma} = 57^\circ 14' 14''$$

- Postavljanje hipoteza

H_0 : Trougao je pravougli

H_a : Trougao nije pravougli

$$H_0: M[\beta - 90^\circ] = 0$$

$$H_a: M[\beta - 90^\circ] \neq 0$$

•Matrica koeficijenata H

$$H = \begin{vmatrix} 0 & 1 \end{vmatrix}$$

•Vektor odstupanja d

$$d = \begin{vmatrix} 8 \end{vmatrix}$$

•Matrica Q_d $Q_d = H Q_x H^T$

$$Q_d = \begin{vmatrix} 0,666667 \end{vmatrix}$$

•Matrica Q_d^{-1}

$$Q_d^{-1} = \begin{vmatrix} 1,5 \end{vmatrix}$$

•Test statistika

$$T = \frac{d^T Q_d^{-1} d}{k \sigma_0^2} \approx F_{0,95}(k, \infty)$$

k – rang matrice H , $F_{0,95}(k, \infty)$ – kvantil Fišerovog rasporeda

$$k = \text{rang}(H) = 1$$

$$T = 3.173354 > F_{0,95}(1, \infty) = 3.841552$$

ZAKLJUČAK:

Alternativna hipoteza H_a se odbacuje. Prihvata se nulta hipoteza H_0 : trougao je pravougli.