1. **PRVA ITERACIJA**

• Direkcioni uglovi i dužine

$ν\_{T,0}^{1}=$ $D\_{1,0}= m$

$ν\_{T,0}^{2}=$ $D\_{2,0}= m$

$ν\_{T,0}^{3}=$ $D\_{3,0}= m$

$ν\_{T,0}^{4}=$ $D\_{4,0}= m$

• Orijentacioni ugao

$$Z\_{0,1}=α\_{1}-ν\_{T,0}^{1}=$$

$Z\_{0,2}=α\_{2}-ν\_{T,0}^{2}=$ $Z\_{O}=\frac{Z\_{0,1}+Z\_{0,2}+Z\_{0,3}+Z\_{0,4}}{4}=$

$$Z\_{0,3}=α\_{3}-ν\_{T,0}^{3}=$$

$$Z\_{0,4}=α\_{4}-ν\_{T,0}^{4}=$$

• Jednačine popravaka

$v\_{D\_{1}}=A\_{1}△x+B\_{1}△y+f\_{D\_{1}}$ $f\_{D\_{1}}=D\_{1,0}-D\_{1}$

$v\_{D\_{2}}=A\_{2}△x+B\_{2}△y+f\_{D\_{2}}$ $f\_{D\_{2}}=D\_{2,0}-D\_{2}$ $A\_{i}=-\cos(ν\_{i})$

$v\_{D\_{3}}=A\_{3}△x+B\_{3}△y+f\_{D\_{3}}$ $f\_{D\_{3}}=D\_{3,0}-D\_{3}$ $ B\_{i}=-\sin(ν\_{i})$

$v\_{D\_{4}}=A\_{4}△x+B\_{4}△y+f\_{D\_{4}}$ $f\_{D\_{4}}=D\_{4,0}-D\_{4}$

$v\_{α\_{1}}=a\_{1}△x+b\_{1}△y+c\_{1}△z+f\_{α\_{1}}$ $f\_{α\_{1}}=\left(ν\_{T,0}^{1}+Z\_{0}\right)-α\_{1}$

$v\_{α\_{2}}=a\_{2}△x+b\_{2}△y+c\_{2}△z+f\_{α\_{2}}$ $f\_{α\_{2}}=\left(ν\_{T,0}^{2}+Z\_{0}\right)-α\_{2}$ $a\_{i}=\frac{ρ''\sin(ν\_{i})}{D\_{i,0}}$

$v\_{α\_{3}}=a\_{3}△x+b\_{3}△y+c\_{3}△z+f\_{α\_{3}}$ $f\_{α\_{3}}=\left(ν\_{T,0}^{3}+Z\_{0}\right)-α\_{3}$ $b\_{i}=-\frac{ρ''\cos(ν\_{i})}{D\_{i,0}}$

$v\_{α\_{4}}=a\_{4}△x+b\_{4}△y+c\_{4}△z+f\_{α\_{4}}$ $f\_{α\_{4}}=\left(ν\_{T,0}^{4}+Z\_{0}\right)-α\_{4}$

• Jednačine popravaka u matričnom obliku

$$\hat{V}=A\hat{x}+f$$

• Matrica dizajna $A$ i vektor slobodnih članova $f$

|  |  |  |  |
| --- | --- | --- | --- |
|  | X | Y | Z |
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|  |  |  |  |
|  |  |  |  |
|  $ A=$ |  |  |  |
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| --- | --- |
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|   |  |
|  $f=$ |  |
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• Homogenizacija težina

$P\_{α\_{i}}=\frac{σ\_{0}^{2}}{σ\_{α\_{i}}^{2}}=\frac{c}{σ\_{α\_{i}}^{2}} , σ\_{0}^{2}=3^{2}=9$ $P\_{D\_{i}}=\frac{σ\_{0}^{2}}{σ\_{D\_{i}}^{2}}=\frac{c}{σ\_{D\_{i}}^{2}} , σ\_{D\_{i}}=5mm+5ppm$

$P\_{α\_{1}}=\frac{9}{9}=1\left[\frac{1}{''^{2}}\right]$ $σ\_{D\_{1}}=5+5∙=mm$ , $P\_{D\_{1}}=\frac{}{^{2}}=[\frac{1}{mm^{2}}]$

$P\_{α\_{2}}=\frac{9}{9}=1\left[\frac{1}{''^{2}}\right]$ $σ\_{D\_{2}}=5+5∙=mm$ , $P\_{D\_{2}}=\frac{}{^{2}}=[\frac{1}{mm^{2}}]$

$P\_{α\_{3}}=\frac{9}{9}=1\left[\frac{1}{''^{2}}\right]$ $σ\_{D\_{3}}=5+5∙=mm$ , $P\_{D\_{3}}=\frac{}{^{2}}=[\frac{1}{mm^{2}}]$

$P\_{α\_{4}}=\frac{9}{9}=1\left[\frac{1}{''^{2}}\right]$ $σ\_{D\_{4}}=5+5∙=mm$ , $P\_{D\_{4}}=\frac{}{^{2}}=[\frac{1}{mm^{2}}]$

• Matrica težina $P$

$$P\_{diag}=[ ]$$

• Matrica koeficijenata normalnih jednacina $N$ i vektor slobodnih članova normalnih jednačina $n$

 $ N=A^{T}∙P∙ A$ $n=A^{T}∙P∙f$

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| --- | --- | --- | --- |
|  |  |  |  |
|  N= |  |  |  |
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| --- | --- |
|  |  |
|  n= |  |
|  |  |

• Vektor nepoznatih parametara$ \hat{x}$ i matrica kofaktora $Q\_{\hat{x}}$

$$\hat{x}=-N^{-1}∙n=-Q\_{\hat{x}}∙n$$

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  $Q\_{\hat{x}}=$ |  |  |  |
|   |  |  |  |

|  |  |
| --- | --- |
|  | $$mm$$ |
|  $\hat{x}=$ | $$mm$$ |
|  | $$''$$ |

• Vektor popravaka merenih veličina $\hat{V}$

$$\hat{V}=A∙\hat{x}+f$$

|  |  |
| --- | --- |
|  | $$mm$$ |
|   | $$mm$$ |
|  | $$mm$$ |
|  $\hat{V}=$ | $$mm$$ |
|   | $$''$$ |
|  | $$''$$ |
|  | $$''$$ |
|  | $$ ''$$ |

• Kontrola izravnanja

 $\hat{V}^{T}∙P∙V=$ $f^{T}∙P∙f+n^{T}∙\hat{x}=$

• Aposteriori standardna devijacija

 $S\_{0}=\sqrt{\frac{\hat{V}^{T}∙P∙V}{f}} , f=n-u$ $S\_{0}=\sqrt{\frac{}{}}=$

• Globalni test na grube greške

* Hipoteze

 $H\_{0}: σ^{2}=σ\_{0}^{2}$ , $H\_{a}: σ^{2}\ne σ\_{0}^{2}$ , $σ=S\_{0}$

* Test statistika

 $F=\frac{S\_{0}^{2}}{σ\_{0}^{2}}= > F\left(0.05,5,\infty \right)=$

 => Prihavata se $H\_{a}$

 Ima grubih grešaka!!

• Data snooping test

 $Q\_{\hat{V}}=P^{-1}∙A∙N^{-1}∙A^{T}$ $t\_{i}=\frac{\hat{v\_{i}}}{σ\_{0}\sqrt{Q\_{\hat{vi}}}}$

$$Q\_{\hat{V},diag}=[ ]$$

 $|t\_{1}|= < t\left(0.05,5\right)=$

 $|t\_{2}|= < t\left(0.05,5\right)=$

 $|t\_{3}|=< t\left(0.05,5\right)=$

 $|t\_{4}|=< t\left(0.05,5\right)=$

 $|t\_{5}|=>t\left(0.05,5\right)=$

 $ |t\_{6}|=< t\left(0.05,5\right)=$

 $|t\_{7}|=< t\left(0.05,5\right)=$

 $|t\_{8}|=< t\left(0.05,5\right)=$

 Odstranimo merenje koje najviše odstupa od vrednosti $ t\left(0.05,5\right)=$ i ponovimo postupak izravnanja.

**DRUGA ITERACIJA**

• Matrica dizajna $A$ i vektor slobodnih članova $f$

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|  $A=$ |  |  |  |
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|  $ f=$ |  |
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• Orijentacioni ugao

$Z\_{0,2}=α\_{2}-ν\_{T,0}^{2}=$

$Z\_{0,3}=α\_{3}-ν\_{T,0}^{3}=$ $Z\_{O}=\frac{Z\_{0,2}+Z\_{0,3}+Z\_{0,4}}{3}=$

$$Z\_{0,4}=α\_{4}-ν\_{T,0}^{4}=$$

• Matrica težina $P$

$$P\_{diag}=[ ]$$

• Matrica koeficijenata normalnih jednacina $N$ i vektor slobodnih članova normalnih jednačina $n$

 $ N=A^{T}∙P∙ A$ $n=A^{T}∙P∙f$

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| --- | --- | --- | --- |
|  |  |  |  |
| $$ N=$$ |  |  |  |
|  |  |  |  |

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|  |  |
|  $n=$ |  |
|  |  |

• Vektor nepoznatih parametara$ \hat{x}$

 $\hat{x}=-N^{-1}∙n=-Q\_{\hat{x}}∙n$

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| --- | --- | --- | --- |
|  |  |  |  |
| $$ Q\_{\hat{x}}=$$ |  |  |  |
|  |  |  |  |

|  |  |
| --- | --- |
|  | $$mm$$ |
|  $\hat{x}=$ | $$mm$$ |
|  | $$''$$ |

• Vektor popravaka merenih veličina $\hat{V}$

$$\hat{V}=A∙\hat{x}+f$$

|  |  |
| --- | --- |
|  | $$mm$$ |
|  | $$mm$$ |
|  | $$mm$$ |
|  $\hat{V}=$ | $$mm$$ |
|  | $$''$$ |
|  | $$''$$ |
|  | $$ ''$$ |

• Kontrola izravnanja

 $\hat{V}^{T}∙P∙V=$ $f^{T}∙P∙f+n^{T}∙\hat{x}=$

• Aposteriori standardna devijacija

 $S\_{0}=\sqrt{\frac{\hat{V}^{T}∙P∙V}{f}} , f=n-u$ $S\_{0}=\sqrt{\frac{}{}}=''$

• Globalni test na grube greške

* Hipoteze

 $H\_{0}: σ^{2}=σ\_{0}^{2}$ , $H\_{a}: σ^{2}\ne σ\_{0}^{2}$ , $σ=S\_{0}$

* Test statistika

 $F=\frac{S\_{0}^{2}}{σ\_{0}^{2}}= < F\left(0.05,4,\infty \right)=$

 => Prihavata se $H\_{0}$

 Nema grubih grešaka.

• Izravnate vrednosti nepoznatih parametara

$$\hat{Y}\_{T}=Y\_{T,0}+\hat{y}== m$$

$$\hat{X}\_{T}=X\_{T,0}+\hat{x}==m$$

$$\hat{Z}=Z\_{0}+\hat{z}=+=$$

• Dužine i pravci iz izravnatih koordinata i izravnatog orijentacionog ugla

 $\hat{D}\_{1}= m$ , $\hat{α}\_{2}=ν\_{T}^{2}+\hat{Z}=+=$

 $\hat{D}\_{2}=m$ , $\hat{α}\_{3}=ν\_{T}^{3}+\hat{Z}=+=$

 $\hat{D}\_{3}=m$ ,$ \hat{α}\_{4}=ν\_{T}^{4}+\hat{Z}=+=$

 $\hat{D}\_{4}= m$

• Definitivna kontrola izravnanja

 $U-\hat{V}=0$ , $U=F\_{i}\left(\hat{X},\hat{Y,}\hat{Z}\right)-l\_{i}$

|  |  |
| --- | --- |
|  | $ \hat{D}\_{1}-D\_{1}$  |
|  | $$\hat{D}\_{2}-D\_{2}$$ |
|  | $$\hat{D}\_{3}-D\_{3}$$ |
|  $ U=$ | $$ \hat{D}\_{4}-D\_{4}$$ |
|  | $$\hat{α}\_{2}-α\_{2}$$ |
|  |  $\hat{α}\_{3}-α\_{3}$ |
|  |  $\hat{α}\_{4}-α\_{4}$ |

|  |  |
| --- | --- |
|   | $$mm$$ |
|   | $$mm$$ |
|   | $$mm$$ |
|  $ U=$ | $$mm$$ |
|   | $$''$$ |
|   | $$''$$ |
|   | $$''$$ |

|  |  |
| --- | --- |
|   | $$mm$$ |
|   | $$mm$$ |
|   | $$mm$$ |
|  $ U-V=$ | $$mm$$ |
|   | $$''$$ |
|   | $$''$$ |
|   | $$''$$ |

• Ocena tačnosti
$$K\_{\hat{x}}=σ\_{0}∙Q\_{\hat{x}}$$

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|  |  |  |  |
|  $K\_{\hat{x}}=$ |  |  |  |
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|  |  |  |  |
| $$ Q\_{\hat{x}}=$$ |  |  |  |
|  |  |  |  |

 $ S\_{\hat{y}}=S\_{0}∙\sqrt{Q\_{yy}} = mm$ , $S\_{\hat{x}}=S\_{0}∙\sqrt{Q\_{xx}}= mm$
$$S\_{p}=\sqrt{ S\_{\hat{y}}^{2}+S\_{\hat{x}}^{2} }= mm$$

• Elipsa grešaka

2)

• Približne vrednosti visina repera

Usvojimo $H\_{R3}= m$

$H\_{R1}^{0}=H\_{R3}-h\_{3}=m$ , $H\_{R2}^{0}=H\_{R3}-h\_{2}= m$

• Jednačine popravaka

$h\_{1}=H\_{R2}-H\_{R1}$ , $h\_{2}=H\_{R3}-H\_{R2}$ , $h\_{3}=H\_{R3}-H\_{R1}$

$v\_{1}=-∆H\_{R1}+∆H\_{R2}+f\_{1}$ , $f\_{1}=\left(H\_{R2}^{0}-H\_{R1}^{0}\right)-h\_{1}$

$v\_{2}=-∆H\_{R2}+f\_{2}$ , $f\_{2}=\left(H\_{R3}-H\_{R2}^{0}\right)-h\_{2}$

$v\_{3}=-∆H\_{R1}+f\_{3}$ , $f\_{3}=\left(H\_{R3}-H\_{R1}^{0}\right)-h\_{1}$

• Jednačine popravaka u matričnom obliku

$$\hat{V}=A\hat{x}+f$$

• Matrica dizajna $A$ i vektor slobodnih članova $f$

|  |  |  |
| --- | --- | --- |
|  | $$H\_{R1}$$ | $$H\_{R2}$$ |
|  |  |  |
| $$ A=$$ |  |  |
|  |  |  |

|  |  |
| --- | --- |
|  |  |
| $$ f=$$ |  |
|  |  |

• Homogenizacija težina

$$P\_{i}=\frac{1}{n\_{i}} , n\_{i}-broj stanica$$

 $P\_{1}=$, $P\_{2}=$, $ P\_{3}=$

• Matrica težina $P$

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| $$ P=$$ |  |  |  |
|  |  |  |  |

• Matrica koeficijenata normalnih jednacina $N$ i vektor slobodnih članova normalnih jednačina $n$

 $ N=A^{T}∙P∙ A$ $n=A^{T}∙P∙f$

|  |  |  |
| --- | --- | --- |
|  |  |  |
| $$ N=$$ |  |  |

|  |  |
| --- | --- |
|  |  |
| $$ n=$$ |  |

• Vektor nepoznatih parametara$ \hat{x}$ i matrica kofaktora $Q\_{\hat{x}}$

$$\hat{x}=-N^{-1}∙n=-Q\_{\hat{x}}∙n$$

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  $Q\_{\hat{x}}=$ |  |  |

|  |  |
| --- | --- |
|  | $$mm$$ |
|  $ \hat{x}=$ | $$mm$$ |

• Vektor popravaka merenih veličina $\hat{V}$

$$\hat{V}=A∙\hat{x}+f$$

|  |  |
| --- | --- |
|  | $$mm$$ |
|  $\hat{V}=$ | $$mm$$ |
|  | $$mm$$ |

• Kontrola izravnanja

$\hat{V}^{T}∙P∙V=$ $f^{T}∙P∙f+n^{T}∙\hat{x}=$

• Aposteriori standardna devijacija

 $S\_{0}=\sqrt{\frac{\hat{V}^{T}∙P∙V}{f}} , f=n-u$ $S\_{0}=\sqrt{\frac{}{}}=$

• Izravnate vrednosti nepoznatih parametara i merenih veličina

 $\hat{H}\_{R1}= m$ , $\hat{H}\_{R2}=m$

 $\hat{h}\_{1}= m$ , $\hat{h}\_{2}= m$ , $\hat{h}\_{3}= m$

• Ocena tačnosti

 $S\_{H\_{R1}}=S\_{0}∙\sqrt{Q\_{xx}}= mm$ *,* $S\_{H\_{R2}}=S\_{0}∙\sqrt{Q\_{yy}}= mm$