

FAKULTET TEHNIČKIH NAUKA
GEODEZIJA I GEOINFORMATIKA

RAČUN IZRAVNANJA
- VEŽBA 5 -

NOVI SAD, 2024.

Izravnanje po metodi posrednih merenja

- Gaus-Markovljev model izravnanja

$$\mathbf{v} = \mathbf{A}\mathbf{x} + \mathbf{f} \quad - \text{funkcionalni model}$$

$$\mathbf{P}_l = \mathbf{Q}_l^{-1}, E(\mathbf{v}) = \mathbf{0} \quad - \text{stohastički model}$$

- Funkcionalnim modelom je definisana funkcionalna (matematička) veza između merenih veličina i nepoznatih parametara modela.
- Stohastički model definiše određene pretpostavke u vezi sa merenjima.

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

U trouglu su mereni uglovi i dužine: α , β , γ , a , b i c . Standard merenja pravaca iznosi 2", a dužina 3mm + 3mm/km.
Izravnati rezultate merenja i oceniti tačnost merenja i izravnatih veličina.

Funkcije veze:

$$\hat{\alpha} = \alpha + v_{\alpha} = X$$

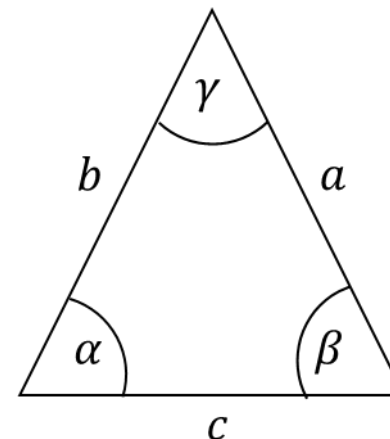
$$\hat{\beta} = \beta + v_{\beta} = Y$$

$$\hat{\gamma} = \gamma + v_{\gamma} = 180^{\circ} - (X + Y)$$

$$\hat{a} = a + v_a = Z$$

$$\hat{b} = b + v_b = \frac{Z}{\sin(X)} \cdot \sin(Y)$$

$$\hat{c} = c + v_c = \frac{Z}{\sin(X)} \cdot \sin(180^{\circ} - (X + Y))$$



Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

Neke od ovih funkcija linearizuju se razvojem u Tejlorov red u okolini približnih vrednosti nepoznatih parametara, nakon čega se dobijaju jednačine popravaka:

$$v_\alpha = dX + f_\alpha, \quad f_\alpha = X_0 - \alpha$$

$$v_\beta = dY + f_\beta, \quad f_\beta = Y_0 - \beta$$

$$v_\gamma = -dX - dY + f_\gamma, \quad f_\gamma = (180^\circ - (X_0 + Y_0)) - \gamma$$

$$v_a = dZ + f_a, \quad f_a = Z_0 - a$$

$$v_b = a_1 dX + b_1 dY + c_1 dZ + f_b, \quad f_b = \left(\frac{Z_0}{\sin(X_0)} \cdot \sin(Y_0) \right) - b$$

$$v_c = a_2 dX + b_2 dY + c_2 dZ + f_c, \quad f_c = \left(\frac{Z_0}{\sin(X_0)} \cdot \sin(180^\circ - (X_0 + Y_0)) \right) - c$$

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

Koeficijenti a_i , b_i i c_i formiraju se na sledeći način:

$$a_1 = \left(\frac{\partial b}{\partial X} \right)_0 = \frac{-Z_0 \cdot \sin Y_0 \cdot \cos X_0}{\sin^2 X_0} \cdot \frac{1}{\rho}$$

$$b_1 = \left(\frac{\partial b}{\partial Y} \right)_0 = \frac{Z_0 \cdot \cos Y_0}{\sin X_0} \cdot \frac{1}{\rho}$$

$$c_1 = \left(\frac{\partial b}{\partial Z} \right)_0 = \frac{\sin Y_0}{\sin X_0}$$

$$a_2 = \left(\frac{\partial c}{\partial X} \right)_0 = \frac{-Z_0 \cdot \cos[180^\circ - (X_0 + Y_0)] \cdot \sin X_0 - Z_0 \cdot \sin[180^\circ - (X_0 + Y_0)] \cdot \cos X_0}{\sin^2 X_0} \cdot \frac{1}{\rho}$$

$$b_2 = \left(\frac{\partial c}{\partial Y} \right)_0 = \frac{Z_0}{\sin X_0} \cdot (-\cos[180^\circ - (X_0 + Y_0)]) \cdot \frac{1}{\rho}$$

$$c_2 = \left(\frac{\partial c}{\partial Z} \right)_0 = \frac{\sin(X_0 + Y_0)}{\sin(X_0)}$$

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

Formiranje matrice dizajna **A** i vektora slobodnih članova **f**:

$$\mathbf{A} = \begin{matrix} & dX & dY & dZ \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} & \begin{matrix} \alpha \\ \beta \\ \gamma \\ a \\ b \\ c \end{matrix} \end{matrix} \quad \mathbf{f} = \begin{bmatrix} f_\alpha \\ f_\beta \\ f_\gamma \\ f_a \\ f_b \\ f_c \end{bmatrix}$$

Napomene:

- Slobodna članove za horizontalne pravce izraziti u sekundama.
- Slobodne članove za dužine izraziti u milimetrima.
- Pri računanju koeficijenata a_i, b_i i c_i dužine Z_0 izraziti u milimetrima.

Stohastički model

- **Primer 1 – dvodimenzionalna geodetska mreža**

Formiranje matrice težina \mathbf{P} :

Težine merenja P_i predstavljaju stepen poverenja u rezultate merenja.

$$\sigma_p = 2'' \Rightarrow \sigma_\alpha = \sigma_\beta = \sigma_\gamma = \sqrt{2} \cdot 2''$$

Za σ_0 usvojiti vrednost **1!**

$$\sigma_a = 3 \text{ mm} + 3 \frac{\text{mm}}{\text{km}} \cdot a [\text{km}]$$

$$\sigma_b = 3 \text{ mm} + 3 \frac{\text{mm}}{\text{km}} \cdot b [\text{km}]$$

$$\sigma_c = 3 \text{ mm} + 3 \frac{\text{mm}}{\text{km}} \cdot c [\text{km}]$$

$$P_\alpha = \frac{\sigma_0^2}{\sigma_{\sigma_\alpha}^2}, P_a = \frac{\sigma_0^2}{\sigma_a^2}$$

$$P_\beta = \frac{\sigma_0^2}{\sigma_{\sigma_\beta}^2}, P_b = \frac{\sigma_0^2}{\sigma_b^2}$$

$$P_\gamma = \frac{\sigma_0^2}{\sigma_{\sigma_\gamma}^2}, P_c = \frac{\sigma_0^2}{\sigma_c^2}$$

$$\mathbf{P} = \begin{bmatrix} P_\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & P_\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & P_\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & P_a & 0 & 0 \\ 0 & 0 & 0 & 0 & P_b & 0 \\ 0 & 0 & 0 & 0 & 0 & P_c \end{bmatrix} \begin{matrix} \alpha \\ \beta \\ \gamma \\ a \\ b \\ c \end{matrix}$$

Kompletno rešenje primera 1 dostupno je u fajlu *Račun izravnjanja - Vežba 5.xlsx*.

Primena metoda najmanjih kvadrata (MNK)

- Sistem normalnih jednačina

$$\mathbf{N}\hat{\mathbf{x}} + \mathbf{n} = \mathbf{0}$$

$$\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A}$$

$$\mathbf{n} = \mathbf{A}^T \mathbf{P} \mathbf{f}$$

- Ocena nepoznatih parametara i popravaka merenih veličina

$$\hat{\mathbf{x}} = -\mathbf{Q}_{\hat{\mathbf{x}}} \cdot \mathbf{n}, \quad \mathbf{Q}_{\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

$$\hat{\mathbf{v}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}$$

Kontrola računanja:

$$\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}} \equiv \mathbf{f}^T \mathbf{P} \mathbf{f} + \mathbf{n}^T \hat{\mathbf{x}}$$

- Ocena disperzionog koeficijenta

$$m_0^2 = \frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f} \quad f = n - u, \quad n - \text{broj merenja}, \quad u - \text{broj nepoznatih parametara}$$

Globlani test na grube greške

- Hipoteze

$$H_0: \sigma^2 = \sigma_0^2 \text{ protiv } H_a: \sigma^2 \neq \sigma_0^2,$$

pri čemu je $\sigma^2 = M(m_0^2)$, a M operator matematičkog očekivanja.

- Test statistika

$$T = \frac{m_0^2}{\sigma_0^2} \sim F_{1-\alpha, f, \infty}$$

Excel: $F_{1-\alpha, f, \infty} \rightarrow \text{FINV}(\alpha, f, 10000)$

Ukoliko je $T < F_{1-\alpha, f, \infty}$, nulta hipoteza se ne odbacuje, tj. nema grubih grešaka.

Ukoliko je $T \geq F_{1-\alpha, f, \infty}$, nulta hipoteza se odbacuje, pa konstatujemo da su u merenjima prisutne grube greške.

Izravnete vrednosti merjenja i nepoznatih parametara

- Izravnete vrednosti nepoznatih parametara

$$\begin{aligned}\hat{X} &= X_0 + \widehat{dX} \\ \hat{Y} &= Y_0 + \widehat{dY} \\ \hat{Z} &= Z_0 + \widehat{dZ}\end{aligned} \quad \hat{\mathbf{x}} = \begin{bmatrix} \widehat{dX} \\ \widehat{dY} \\ \widehat{dZ} \end{bmatrix}$$

- Izravnete vrednosti merenih veličina

$$\begin{aligned}\hat{\alpha} &= \alpha + \hat{v}_\alpha \\ \hat{\beta} &= \beta + \hat{v}_\beta \\ \hat{\gamma} &= \gamma + \hat{v}_\gamma \\ \hat{a} &= a + \hat{v}_a \\ \hat{b} &= b + \hat{v}_b \\ \hat{c} &= c + \hat{v}_c\end{aligned} \quad \hat{\mathbf{v}} = \begin{bmatrix} \hat{v}_\alpha \\ \hat{v}_\beta \\ \hat{v}_\gamma \\ \hat{v}_a \\ \hat{v}_b \\ \hat{v}_c \end{bmatrix}$$

Analiza tačnosti geodetskih mreža

- *A posteriori* standardna devijacija (globalna mera tačnosti)

$$m_0 = \sqrt{\frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f}},$$

pri čemu je $f = n - u$ broj stepeni slobode.

- Standardne devijacije nepoznatih parametara (lokalne mere tačnosti)

$$\hat{\sigma}_X = m_0 \sqrt{Q_{\hat{X}\hat{X}}},$$

$$\hat{\sigma}_Y = m_0 \sqrt{Q_{\hat{Y}\hat{Y}}},$$

$$\hat{\sigma}_Z = m_0 \sqrt{Q_{\hat{Z}\hat{Z}}},$$

$$\mathbf{Q}_{\hat{\mathbf{x}}} = \begin{bmatrix} Q_{\hat{X}\hat{X}} & Q_{\hat{X}\hat{Y}} & Q_{\hat{X}\hat{Z}} \\ Q_{\hat{Y}\hat{X}} & Q_{\hat{Y}\hat{Y}} & Q_{\hat{Y}\hat{Z}} \\ Q_{\hat{Z}\hat{X}} & Q_{\hat{Z}\hat{Y}} & Q_{\hat{Z}\hat{Z}} \end{bmatrix}$$

pri čemu su $Q_{\hat{X}\hat{X}}$, $Q_{\hat{Y}\hat{Y}}$ i $Q_{\hat{Z}\hat{Z}}$ dijagonalni elementi kofaktorske matrice $\mathbf{Q}_{\hat{\mathbf{x}}}$.

Funkcionalni model

- Jednačine popravaka – visinske razlike (geometrijski nivelman)

$$\widehat{\Delta h}_{i-j} = \Delta h_{i-j} + v_{\Delta h_{i-j}} \quad (1)$$

$$\widehat{\Delta h}_{i-j} = \widehat{H}_j - \widehat{H}_i \quad (2)$$

Na osnovu izraza (1) i (2) može se napisati:

$$v_{\Delta h_{i-j}} = (\widehat{H}_j - \widehat{H}_i) - \Delta h_{i-j}, \quad \widehat{H}_j = H_j^0 + dH_j \quad \text{i} \quad \widehat{H}_i = H_i^0 + dH_i,$$

$$v_{\Delta h_{i-j}} = H_j^0 + dH_j - H_i^0 - dH_i - \Delta h_{i-j},$$

a onda:

$$v_{\Delta h_{i-j}} = dH_j - dH_i + f_{\Delta h_{i-j}}, \quad f_{\Delta h_{i-j}} = (H_j^0 - H_i^0) - \Delta h_{i-j}.$$

Funkcionalni model

• Primer 2 – jednodimenzionalna geodetska mreža

U nivelmanskoj mreži koja se sastoji od 6 repera merene su visinske razlike metodom geometrijskog nivelmana. Izravnati rezultate merenja i oceniti tačnost nepoznatih parametara.

Dati reperi: 10, 11 i 12

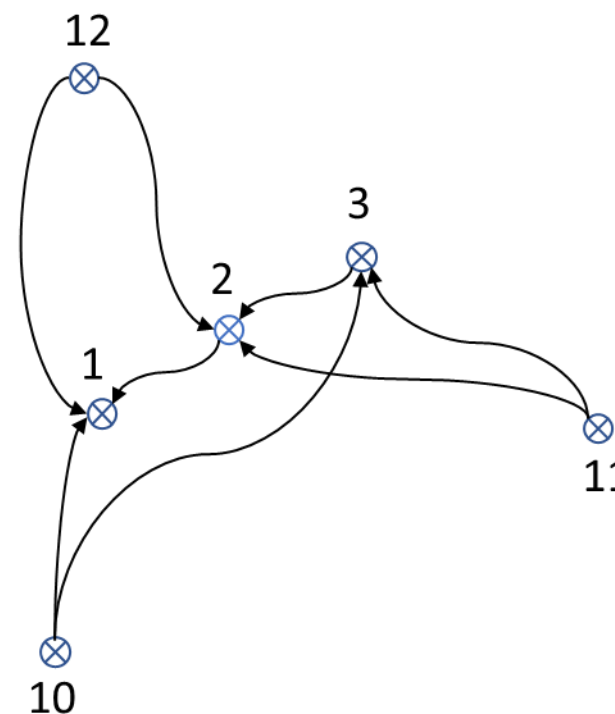
Nepoznati reperi: 1, 2 i 3

Približne visine repera:

$$H_{1,1}^0 = H_{10} + \Delta h_{10-1}, H_{1,2}^0 = H_{12} + \Delta h_{12-1}, H_1^0 = \frac{H_{1,1}^0 + H_{1,2}^0}{2}$$

$$H_{2,1}^0 = H_{11} + \Delta h_{11-2}, H_{2,2}^0 = H_{12} + \Delta h_{12-2}, H_2^0 = \frac{H_{2,1}^0 + H_{2,2}^0}{2}$$

$$H_{3,1}^0 = H_{10} + \Delta h_{10-3}, H_{3,2}^0 = H_{11} + \Delta h_{11-3}, H_3^0 = \frac{H_{3,1}^0 + H_{3,2}^0}{2}$$



Funkcionalni model

- **Primer 2 – jednodimenzionalna geodetska mreža**

Jednačine popravaka:

$$v_{\Delta h_{10-1}} = dH_1 + f_{\Delta h_{10-1}}, \quad f_{\Delta h_{10-1}} = (H_1^0 - H_{10}) - \Delta h_{10-1}$$

$$v_{\Delta h_{10-3}} = dH_3 + f_{\Delta h_{10-3}}, \quad f_{\Delta h_{10-3}} = (H_3^0 - H_{10}) - \Delta h_{10-3}$$

$$v_{\Delta h_{11-2}} = dH_2 + f_{\Delta h_{11-2}}, \quad f_{\Delta h_{11-2}} = (H_2^0 - H_{11}) - \Delta h_{11-2}$$

$$v_{\Delta h_{11-3}} = dH_3 + f_{\Delta h_{11-3}}, \quad f_{\Delta h_{11-3}} = (H_3^0 - H_{11}) - \Delta h_{11-3}$$

$$v_{\Delta h_{12-1}} = dH_1 + f_{\Delta h_{12-1}}, \quad f_{\Delta h_{12-1}} = (H_1^0 - H_{12}) - \Delta h_{12-1}$$

$$v_{\Delta h_{12-2}} = dH_2 + f_{\Delta h_{12-2}}, \quad f_{\Delta h_{12-2}} = (H_2^0 - H_{12}) - \Delta h_{12-2}$$

$$v_{\Delta h_{2-1}} = dH_1 - dH_2 + f_{\Delta h_{2-1}}, \quad f_{\Delta h_{2-1}} = (H_1^0 - H_2^0) - \Delta h_{2-1}$$

$$v_{\Delta h_{3-2}} = dH_2 - dH_3 + f_{\Delta h_{3-2}}, \quad f_{\Delta h_{3-2}} = (H_2^0 - H_3^0) - \Delta h_{3-2}$$

Funkcionalni model

- **Primer 2 – jednodimenzionalna geodetska mreža**

Formiranje matrice dizajna **A** i vektora slobodnih članova **f**:

$$\mathbf{A} = \begin{array}{c} dH_1 \quad dH_2 \quad dH_3 \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & \Delta h_{10-1} \\ 0 & 0 & 1 & \Delta h_{10-3} \\ 0 & 1 & 0 & \Delta h_{11-2} \\ 0 & 0 & 1 & \Delta h_{11-3} \\ 1 & 0 & 0 & \Delta h_{12-1} \\ 0 & 1 & 0 & \Delta h_{12-2} \\ 1 & -1 & 0 & \Delta h_{2-1} \\ 0 & 1 & -1 & \Delta h_{3-2} \end{array} \right] \end{array} \quad \mathbf{f} = \begin{bmatrix} f_{\Delta h_{10-1}} \\ f_{\Delta h_{10-3}} \\ f_{\Delta h_{11-2}} \\ f_{\Delta h_{11-3}} \\ f_{\Delta h_{12-1}} \\ f_{\Delta h_{12-2}} \\ f_{\Delta h_{2-1}} \\ f_{\Delta h_{3-2}} \end{bmatrix}$$

Napomena:

- Slobodna članove izraziti u milimetrima.

Stohastički model

- **Primer 2 – jednodimenzionalna geodetska mreža**

Formiranje težina merenja:

$$P_{\Delta h_{i-j}} = \frac{1}{S_{i-j}},$$

S_{i-j} – dužina nivelmanske strane u kilometrima.

$$P_{\Delta h_{10-1}} = \frac{1}{S_{10-1}}, P_{\Delta h_{12-1}} = \frac{1}{S_{12-1}}$$

$$P_{\Delta h_{10-3}} = \frac{1}{S_{10-3}}, P_{\Delta h_{12-2}} = \frac{1}{S_{12-2}}$$

$$P_{\Delta h_{11-2}} = \frac{1}{S_{11-2}}, P_{\Delta h_{2-1}} = \frac{1}{S_{2-1}}$$

$$P_{\Delta h_{11-3}} = \frac{1}{S_{11-3}}, P_{\Delta h_{3-2}} = \frac{1}{S_{3-2}}$$

$$\mathbf{P} = \begin{bmatrix} P_{\Delta h_{10-1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{\Delta h_{10-3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{\Delta h_{11-2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{\Delta h_{11-3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{\Delta h_{12-1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{12-2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{2-1}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{3-2}} & 0 \end{bmatrix} \begin{bmatrix} \Delta h_{10-1} \\ \Delta h_{10-3} \\ \Delta h_{11-2} \\ \Delta h_{11-3} \\ \Delta h_{12-1} \\ \Delta h_{12-2} \\ \Delta h_{2-1} \\ \Delta h_{3-2} \end{bmatrix}$$

Kompletno rešenje primera 2 dostupno je u fajlu *Račun izravnjanja - Vežba 5.xlsx*.

Primena metoda najmanjih kvadrata (MNK)

- Sistem normalnih jednačina

$$\mathbf{N}\hat{\mathbf{x}} + \mathbf{n} = \mathbf{0}$$

$$\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A}$$

$$\mathbf{n} = \mathbf{A}^T \mathbf{P} \mathbf{f}$$

- Ocena nepoznatih parametara i popravaka merenih veličina

$$\hat{\mathbf{x}} = -\mathbf{Q}_{\hat{\mathbf{x}}} \cdot \mathbf{n}, \quad \mathbf{Q}_{\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

$$\hat{\mathbf{v}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}$$

Kontrola računanja:

$$\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}} \equiv \mathbf{f}^T \mathbf{P} \mathbf{f} + \mathbf{n}^T \hat{\mathbf{x}}$$

- Ocena disperzionog koeficijenta

$$m_0^2 = \frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f} \quad f = n - u, \quad n - \text{broj merenja}, \quad u - \text{broj nepoznatih parametara}$$

Globlani test na grube greške

- Hipoteze

$$H_0: \sigma^2 = \sigma_0^2 \text{ protiv } H_a: \sigma^2 \neq \sigma_0^2,$$

pri čemu je $\sigma^2 = M(m_0^2)$, a M operator matematičkog očekivanja.

- Test statistika

$$T = \frac{m_0^2}{\sigma_0^2} \sim F_{1-\alpha, f, \infty}$$

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Ukoliko je $T \geq F_{1-\alpha, f, \infty}$, nulta hipoteza se odbacuje, pa konstatujemo da su u merenjima prisutne grube greške.

Izravnanje vrednosti merjenja i nepoznatih parametara

- Izravnanje vrednosti nepoznatih parametara

$$\begin{aligned}\widehat{H}_1 &= H_1^0 + d\widehat{H}_1 \\ \widehat{H}_2 &= H_2^0 + d\widehat{H}_2 \\ \widehat{H}_3 &= H_3^0 + d\widehat{H}_3\end{aligned}\quad \widehat{\mathbf{x}} = \begin{bmatrix} d\widehat{H}_1 \\ d\widehat{H}_2 \\ d\widehat{H}_3 \end{bmatrix}$$

- Izravnanje vrednosti merjenih veličina

$$\begin{aligned}\widehat{\Delta h}_{10-1} &= \Delta h_{10-1} + \widehat{v}_{\Delta h_{10-1}} \\ \widehat{\Delta h}_{10-3} &= \Delta h_{10-3} + \widehat{v}_{\Delta h_{10-3}} \\ \widehat{\Delta h}_{11-2} &= \Delta h_{11-2} + \widehat{v}_{\Delta h_{11-2}} \\ \widehat{\Delta h}_{11-3} &= \Delta h_{11-3} + \widehat{v}_{\Delta h_{11-3}} \\ \widehat{\Delta h}_{12-1} &= \Delta h_{12-1} + \widehat{v}_{\Delta h_{12-1}} \\ \widehat{\Delta h}_{12-2} &= \Delta h_{12-2} + \widehat{v}_{\Delta h_{12-2}} \\ \widehat{\Delta h}_{2-1} &= \Delta h_{2-1} + \widehat{v}_{\Delta h_{2-1}} \\ \widehat{\Delta h}_{3-2} &= \Delta h_{3-2} + \widehat{v}_{\Delta h_{3-2}}\end{aligned}\quad \widehat{\mathbf{v}} = \begin{bmatrix} \widehat{v}_{\Delta h_{10-1}} \\ \widehat{v}_{\Delta h_{10-3}} \\ \widehat{v}_{\Delta h_{11-2}} \\ \widehat{v}_{\Delta h_{11-3}} \\ \widehat{v}_{\Delta h_{12-1}} \\ \widehat{v}_{\Delta h_{12-2}} \\ \widehat{v}_{\Delta h_{2-1}} \\ \widehat{v}_{\Delta h_{3-2}} \end{bmatrix}$$

Analiza tačnosti geodetskih mreža

- *A posteriori* standardna devijacija (globalna mera tačnosti)

$$m_0 = \sqrt{\frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f}},$$

pri čemu je $f = n - u$ broj stepeni slobode.

- Standardne devijacije visina repera (lokalne mere tačnosti)

$$\hat{\sigma}_{H_i} = m_0 \sqrt{Q_{\hat{H}_i}}$$

σ_0 - *a priori* standardna devijacija

$Q_{\hat{H}_i}$ - dijagonalni elementi kofaktorske matrice $\mathbf{Q}_{\hat{\mathbf{x}}}$

$$\mathbf{Q}_{\hat{\mathbf{x}}} = \begin{bmatrix} Q_{\hat{H}_1} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & Q_{\hat{H}_m} \end{bmatrix}$$