1)

• Funkcije veze

 Usvojimo $ α=x$ , $β=y$ , $a=z$

 $α^{'}=α+v\_{1}=x$ $a^{'}=a+v\_{4}=z$

 $β^{'}=β+v\_{2}=y$ $b^{'}=b+v\_{5}=\frac{z}{\sin(x)}∙\sin(y)$

 $γ^{'}=γ+v\_{3}=180-(x+y)$ $c^{'}=c+v\_{6}=\frac{z}{\sin(x)}∙\sin((180-\left(x+y\right)))$

• Jednačine popravaka

 $v\_{1}=∆x+f\_{1}$ , $f\_{1}=x-α$

 $v\_{2}=∆y+f\_{2}$ , $f\_{2}=y-β$

 $v\_{3}=-∆x-∆y+f\_{3}$ , $f\_{3}=\left(180-\left(x+z\right)\right)-γ$

 $ v\_{4}=∆z+f\_{4}$ , $f\_{4}=z-a$

 $v\_{5}=a\_{1}∆x+b\_{1}∆y+c\_{1}∆z+f\_{5}$ , $f\_{5}=\left(\frac{z}{\sin(x)}∙\sin(y)\right)-b$

 $v\_{6}=a\_{2}∆x+b\_{2}∆y+c\_{2}∆z+f\_{6}$ , $f\_{6}=(\frac{z}{\sin(x)}∙\sin(\left(180-\left(x+y\right)\right)))-c$

$$ a\_{1}=\frac{∂b}{∂x}=\frac{-z\_{o}∙\sin(y\_{o}∙\cos(x\_{0}))}{sin^{2} x\_{0}}∙\frac{1}{ρ}=$$

$$ b\_{1}=\frac{∂b}{∂y}=\frac{z\_{o}∙\cos(y\_{o})}{sin x\_{0}}∙\frac{1}{ρ}=$$

$$ c\_{1}=\frac{∂b}{∂z}=\frac{\sin(y\_{o})}{sin x\_{0}}=$$

$$ a\_{2}=\frac{∂c}{∂x}=\frac{-z\_{o}∙\cos(\left[180°-\left(x\_{0}+y\_{o}\right)\right])∙\sin(x\_{0}-z\_{o}∙\sin(\left[180°-\left(x\_{0}+y\_{o}\right)\right])∙cosx\_{0})}{sin^{2} x\_{0}}∙\frac{1}{ρ}=$$

$$ b\_{2}=\frac{∂c}{∂y}=\frac{z\_{o}}{sin x\_{0}}∙\left[-\cos([)180°-\left(x\_{0}+y\_{o}\right)\right]=$$

$$ c\_{2}=\frac{∂c}{∂z}=\frac{sin\left(x\_{0}+y\_{o}\right)}{sin x\_{0}}=$$

• Jednačine popravaka u matričnom obliku

$$\hat{V}=A\hat{x}+f$$

• Matrica dizajna $A$ i vektor slobodnih članova $f$

|  |  |  |  |
| --- | --- | --- | --- |
|  | $$x$$ |  $ y$ |  $z$ |
|  |  |  |  |
|  |  |  |  |
| $$ A=$$ |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

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|  |  |
| $$ f=$$ |  |
|  |  |
|  |  |
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• Homogenizacija težina

 $σ\_{p}=3''$ , $P\_{α\_{i}}=\frac{σ\_{0}^{2}}{σ\_{α\_{i}}^{2}}=\frac{c}{σ\_{α\_{i}}^{2}} , σ\_{0}^{2}==$

 $σ\_{α}=3∙\sqrt{2}=''$ , $P\_{α}=\frac{}{}=\left[\frac{1}{''^{2}}\right]$

 $σ\_{β}=3∙\sqrt{2}=''$ , $P\_{β}=\frac{}{}=\left[\frac{1}{''^{2}}\right]$

 $σ\_{γ}=3∙\sqrt{2}=''$ , $P\_{γ}=\frac{}{}=\left[\frac{1}{''^{2}}\right]$

 $σ\_{d}=5mm+5ppm$ , $P\_{D\_{i}}=\frac{σ\_{0}^{2}}{σ\_{D\_{i}}^{2}}=\frac{c}{σ\_{D\_{i}}^{2}} , σ\_{0}^{2}=^{2}=$

 $σ\_{a}=5+5∙\left(\right)= mm$ , $P\_{a}=\frac{}{}=[\frac{1}{mm^{2}}]$

 $σ\_{b}=5+5∙\left(\right)= mm$ , $ P\_{b}=\frac{}{}=[\frac{1}{mm^{2}}]$

 $σ\_{c}=5+5∙\left(\right)= mm$ , $P\_{c}=\frac{}{}=[\frac{1}{mm^{2}}]$

• Matrica težina $P$

$$P\_{diag}=[ ]$$

• Matrica koeficijenata normalnih jednačina $N$ i vektor slobodnih članova normalnih jednačina $n$

 $ N=A^{T}∙P∙ A$ $n=A^{T}∙P∙f$

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| --- | --- | --- | --- |
|  |  |  |  |
| $$ N=$$ |  |  |  |
|  |  |  |  |

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| --- | --- |
|  |  |
| $$ n=$$ |  |
|  |  |

• Vektor nepoznatih parametara$ \hat{x}$ i matrica kofaktora $Q\_{\hat{x}}$

$$\hat{x}=-N^{-1}∙n=-Q\_{\hat{x}}∙n$$

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| --- | --- | --- | --- |
|  |  |  |  |
| $$ Q\_{\hat{x}}=$$ |  |  |  |
|  |  |  |  |

|  |  |
| --- | --- |
|  | $$''$$ |
|  $\hat{x}=$ | $$''$$ |
|  | $$mm$$ |

• Vektor popravaka merenih veličina $\hat{V}$

$$\hat{V}=A∙\hat{x}+f$$

|  |  |
| --- | --- |
|  | $$mm$$ |
|  | $$mm$$ |
|  $\hat{V}=$ | $$mm$$ |
|  | $$''$$ |
|  | $$''$$ |
|  | $$''$$ |

• Kontrola izravnanja

 $\hat{V}^{T}∙P∙V=$ $f^{T}∙P∙f+n^{T}∙\hat{x}=$

• Aposteriori standardna devijacija

 $S\_{0}=\sqrt{\frac{\hat{V}^{T}∙P∙V}{f}} , f=n-u$ $S\_{0}=\sqrt{\frac{}{}}=$

• Globalni test na grube greške

* Hipoteze

 $H\_{0}: σ^{2}=σ\_{0}^{2}$ , $H\_{a}: σ^{2}\ne σ\_{0}^{2}$ , $σ=S\_{0}$

* Test statistika

 $F=\frac{S\_{0}^{2}}{σ\_{0}^{2}}= > F\left(0.05,5,\infty \right)=$

 => Prihavata se $H\_{0}$

 Nema grubih grešaka!!

• Izravnate vrednosti nepoznatih parametara i merenih veličina

 $\hat{x}=$ $\hat{γ}=$

 $\hat{y}=$ $\hat{b}=m$

 $ \hat{z}=m$ $\hat{c}=m$

• Ocena tačnosti
$$K\_{\hat{x}}=σ\_{0}∙Q\_{\hat{x}}$$

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| $$K\_{\hat{x}}=$$ |  |  |  |
|  |  |  |  |

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| --- | --- | --- | --- |
|  |  |  |  |
|  $Q\_{\hat{x}}=$ |  |  |  |
|  |  |  |  |

 $ S\_{\hat{y}}=σ\_{0}∙\sqrt{Q\_{\hat{y}}} =''$

 $S\_{\hat{x}}=σ\_{0}∙\sqrt{Q\_{\hat{x}}}= ''$

$ S\_{\hat{z}}=σ\_{0}∙\sqrt{Q\_{\hat{z}}}= mm$

2)

• Približne visine repera$ H\_{R1}$ i $H\_{R2}$

$$H\_{R1,1}^{0}=H\_{A}+h\_{1 } , H\_{R1,2}^{0}=H\_{B}-h\_{2} H\_{R1}^{0}=\frac{H\_{R1,1}^{0}+H\_{R1,2}^{0}}{2}= m$$

$$H\_{R2,1}^{0}=H\_{B}+h\_{4 } , H\_{R2,2}^{0}=H\_{C}-h\_{5 } H\_{R2}^{0}=\frac{H\_{R2,1}^{0}+H\_{R2,2}^{0}}{2}=m$$

• Funkcije veze

$$h\_{1}+v\_{1}=H\_{R1}-H\_{A}$$

$$h\_{2}+v\_{2}=H\_{B}-H\_{R1}$$

$$h\_{3}+v\_{3}=H\_{R2}-H\_{R1}$$

$$h\_{4}+v\_{4}=H\_{R1}-H\_{B}$$

$$h\_{5}+v\_{5}=H\_{C}-H\_{R2}$$

• Jednačine popravaka

 $v\_{1}= ∆H\_{R1}+f\_{1} $ , $f\_{1}=\left(H\_{R1}^{0}-H\_{A}\right)-h\_{1}$

 $v\_{2}= -∆H\_{R1}+f\_{2}$ , $f\_{2}=\left(H\_{B}-H\_{R1}^{0}\right)-h\_{2}$

 $v\_{3}= -∆H\_{R1}+∆H\_{R2}+f\_{3}$ , $f\_{3}=\left(H\_{R2}^{0}-H\_{R1}^{0}\right)-h\_{3}$

 $v\_{4}= -∆H\_{R1}+f\_{4}$ , $f\_{4}=\left(H\_{R1}^{0}-H\_{B}\right)-h\_{4}$

 $v\_{5}=- ∆H\_{R2}+f\_{5}$ , $f\_{5}=\left(H\_{C}-H\_{R2}^{0}\right)-h\_{5}$

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| --- | --- |
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| $$ f=$$ |  |
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|  |  |

 • Matrica dizajna $A$ i vektor slobodnih članova $f$

|  |  |  |
| --- | --- | --- |
|  |  $H\_{R1}$ |  $H\_{R2}$ |
|  |  |  |
|  |  |  |
| $$ A=$$ |  |  |
|  |  |  |
|  |  |  |

• Homogenizacija težina

$$P\_{h\_{i}}=\frac{1}{S\_{i}}$$

 $P\_{h\_{1}}=\frac{1}{S\_{1}}=[\frac{1}{km}]$ , $P\_{h\_{2}}=\frac{1}{S\_{2}}=[\frac{1}{km}]$ , $P\_{h\_{3}}=\frac{1}{S\_{3}}=[\frac{1}{km}]$

 $P\_{h\_{4}}=\frac{1}{S\_{4}}= [\frac{1}{km}]$ , $P\_{h\_{5}}=\frac{1}{S\_{5}}=[\frac{1}{km}]$

• Matrica težina $P$

$$P\_{diag}=[]$$

• Matrica koeficijenata normalnih jednačina $N$ i vektor slobodnih članova normalnih jednačina $n$

 $ N=A^{T}∙P∙ A$ $n=A^{T}∙P∙f$

|  |  |  |
| --- | --- | --- |
|  |  |  |
| $$ N=$$ |  |  |

|  |  |
| --- | --- |
|  |  |
| $$ n=$$ |  |

• Vektor nepoznatih parametara$ \hat{x}$ i matrica kofaktora $Q\_{\hat{x}}$

$$\hat{x}=-N^{-1}∙n=-Q\_{\hat{x}}∙n$$

|  |  |  |
| --- | --- | --- |
|  |  |  |
| $$Q\_{\hat{x}}=$$ |  |  |

|  |  |
| --- | --- |
|  | $$mm$$ |
| $$\hat{x}=$$ | $$mm$$ |

• Vektor popravaka merenih veličina $\hat{V}$

$$\hat{V}=A∙\hat{x}+f$$

|  |  |
| --- | --- |
|  | $$mm$$ |
|  | $$mm$$ |
| $$\hat{V}=$$ | $$mm$$ |
|  | $$mm$$ |
|  | $$mm$$ |

• Kontrola izravnanja

 $\hat{V}^{T}∙P∙V=$ $f^{T}∙P∙f+n^{T}∙\hat{x}=$

• Aposteriori standardna devijacija

 $S\_{0}=\sqrt{\frac{\hat{V}^{T}∙P∙V}{f}} , f=n-u$ $S\_{0}=\sqrt{\frac{}{}}= $

• Izravnate vrednosti nepoznatih parametara

$$\hat{H}\_{R1}= m$$

$$\hat{H}\_{R2}= m$$

• Ocena tačnosti
$$K\_{\hat{x}}=σ\_{0}∙Q\_{\hat{x}}$$

|  |  |  |
| --- | --- | --- |
|  |  |  |
| $$Q\_{\hat{x}}=$$ |  |  |

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  $K\_{\hat{x}}=$ |  |  |

 $ S\_{H\_{R1}}=S\_{0}∙\sqrt{Q\_{yy}} = mm$ , $S\_{H\_{R2}}=S\_{0}∙\sqrt{Q\_{xx}}=mm$