FAKULTET TEHNIČKIH NAUKA
DEPARTMAN ZA GRAĐEVINARSTVO I GEODEZIJU
LABORATORIJA ZA GEODEZIJU

INŽENJERSKA GEODEZIJA 1

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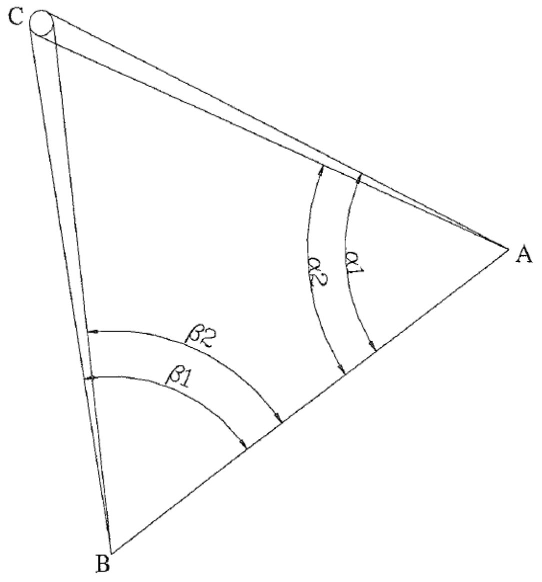
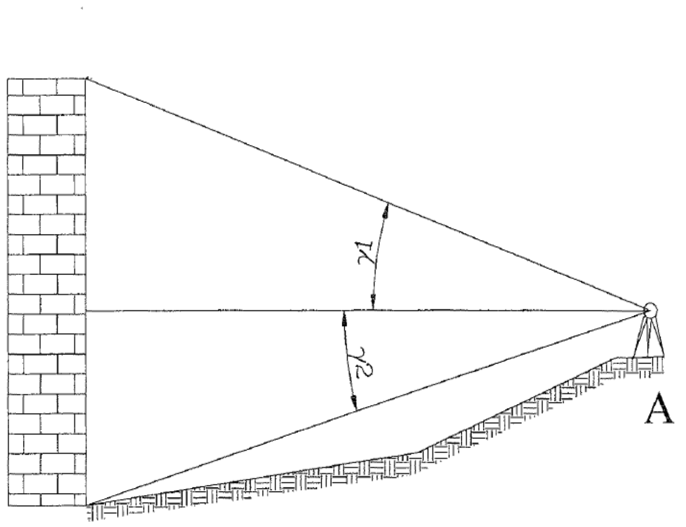
Vežba 2

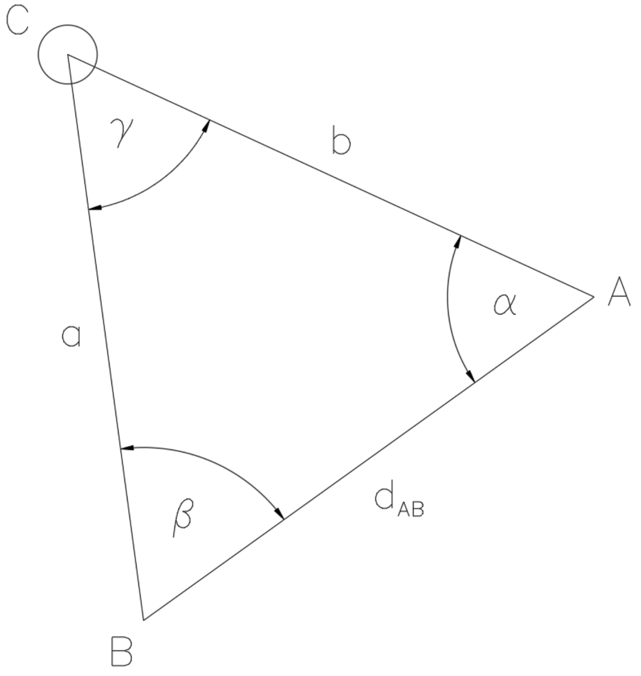
- Sračunati visinu i poluprečnik dimnjaka
- Sračunati sa kojim su standardnim odstupanjem određeni poluprečnik i visina





Vežba 2

- Poluprečnik dimnjaka
  - Potrebno je svesti merene uglove na centar
  - Koristi se aritmetička sredina



$$\alpha = \frac{\alpha_1+\alpha_2}{2} =$$

$$\beta = \frac{\beta_1+\beta_2}{2} =$$

$$\gamma = 180 - (\alpha + \beta) =$$

$$d_{A-B} = \sqrt{(Y_B - Y_A)^2 + (X_B - X_A)^2} =$$

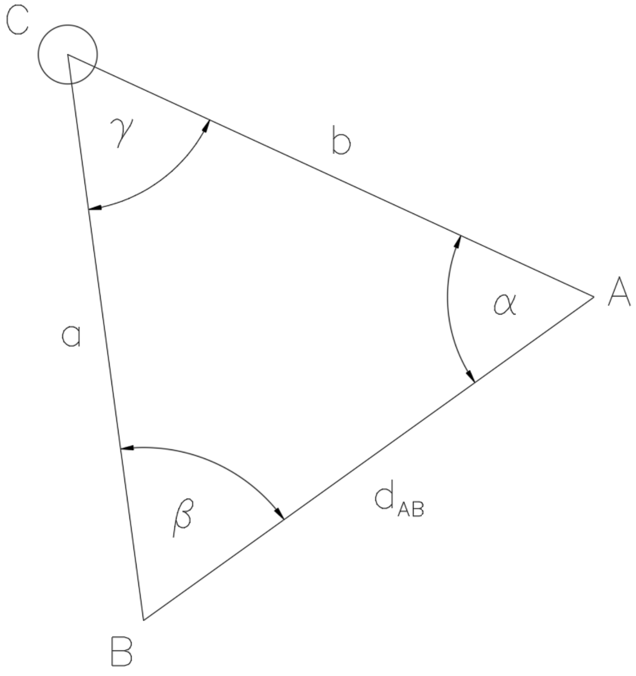
- Sinusna teorema – dužine a, b

$$a = \frac{d_{A-B}}{\sin\gamma}\sin\alpha =$$

$$b = \frac{d_{A-B}}{\sin\gamma}\sin\beta =$$

Vežba 2

- Poluprečnik dimnjaka
  - Potrebno je svesti merene uglove na centar
  - Koristi se aritmetička sredina



$$\alpha = \frac{\alpha_1+\alpha_2}{2} = 60°\,03'\,01''$$
$$\beta = \frac{\beta_1+\beta_2}{2} = 62°\,05'\,24''$$
$$\gamma = 180 - (\alpha + \beta) = 57°\,51'\,35''$$
$$d_{A-B} = \sqrt{(Y_B - Y_A)^2 + (X_B - X_A)^2} = 109.211m$$

- Sinusna teorema – dužine a, b

$$a = \frac{d_{A-B}}{\sin\gamma}\sin\alpha = 111.75m$$
$$b = \frac{d_{A-B}}{\sin\gamma}\sin\beta = 113.97m$$

Vežba 2

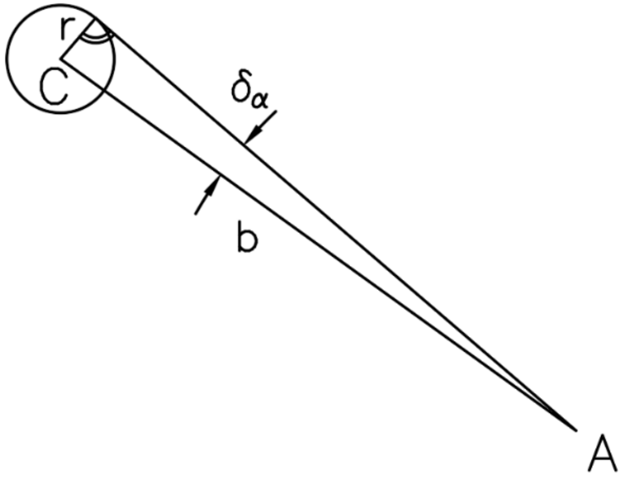
- Poluprečnik dimnjaka
  - Rešavanjem pomoćnih trouglova – poluprečnik r

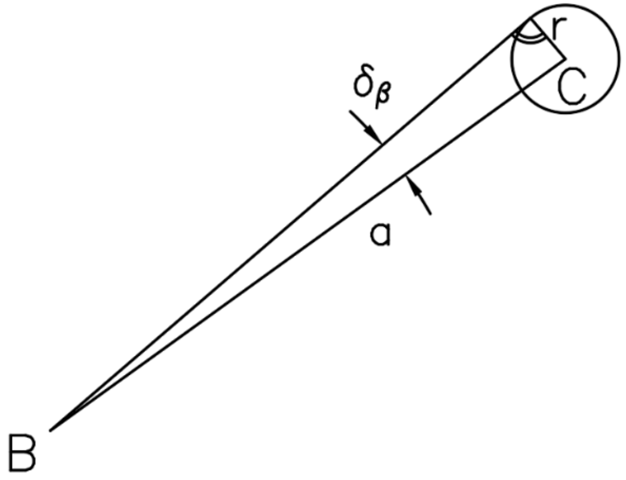
$$\delta_\alpha = \frac{\alpha_1 - \alpha_2}{2} =$$

$$r = b \cdot \sin \delta_\alpha =$$

$$\delta_\beta = \frac{\beta_1 - \beta_2}{2} =$$

$$r = a \cdot \sin \delta_\beta =$$





Vežba 2

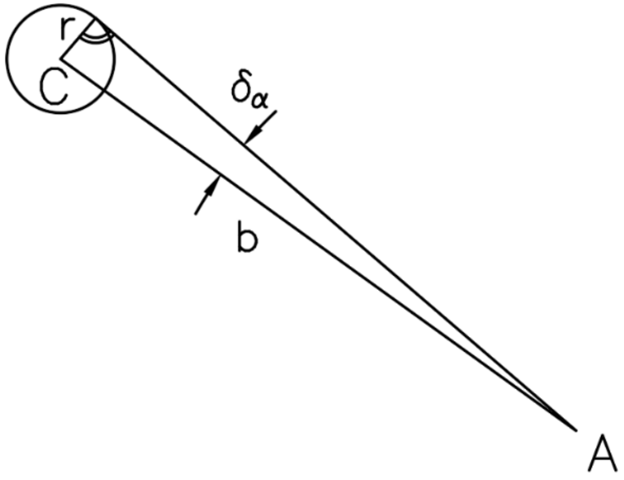
- Poluprečnik dimnjaka
  - Rešavanjem pomoćnih trouglova – poluprečnik r

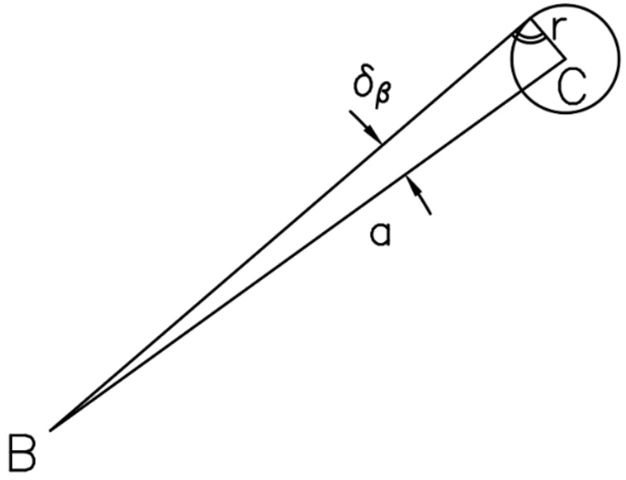
$$\delta_\alpha = \frac{\alpha_1 - \alpha_2}{2} = 0^\circ 20' 41''$$

$$r = b \cdot \sin \delta_\alpha = 0.686m$$

$$\delta_\beta = \frac{\beta_1 - \beta_2}{2} = 0^\circ 21' 06''$$

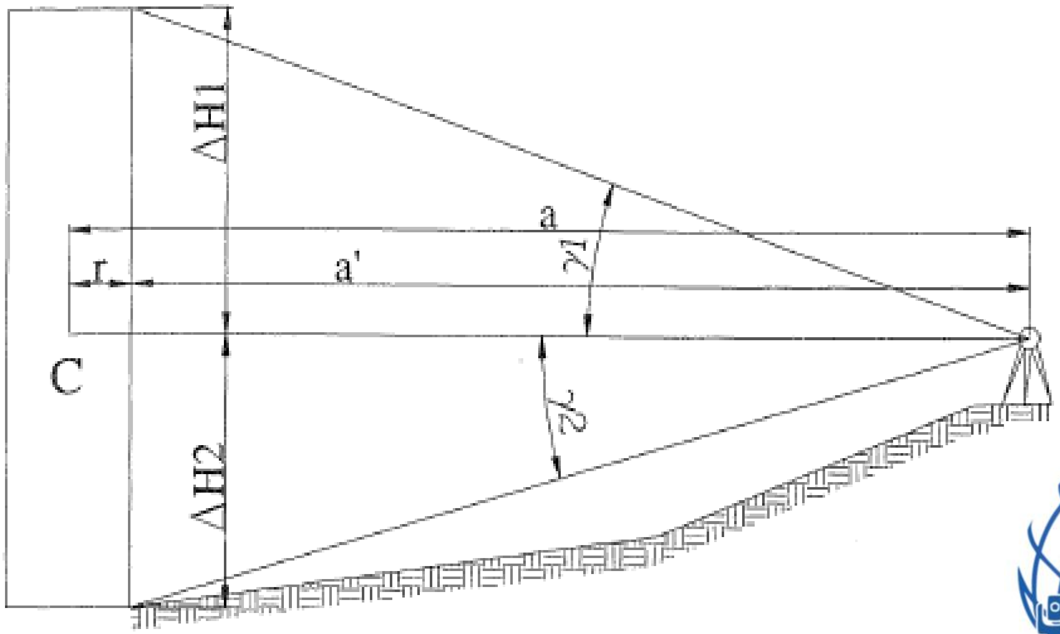
$$r = a \cdot \sin \delta_\beta = 0.686m$$





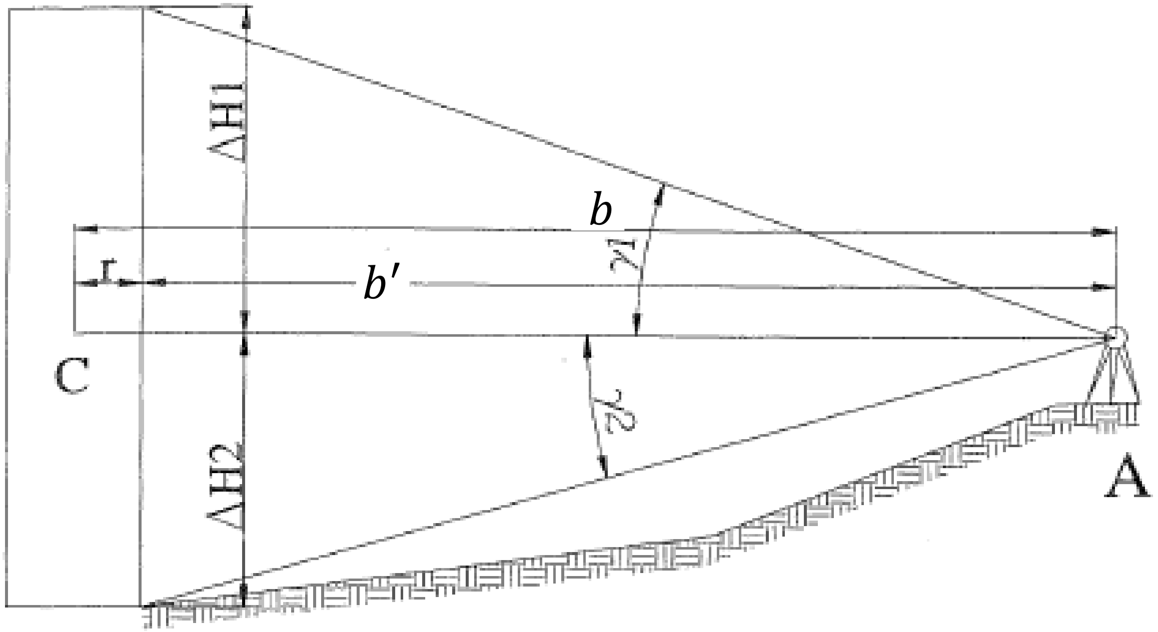
Vežba 2

- Visina dimnjaka
  - Potrebno svesti merenja na centar: $b' = b - r$
  - Visina dimnjaka biće: $H = \Delta H_1 + \Delta H_2 = b' \cdot tg\gamma_1 + b' \cdot tg\gamma_2$
  - Iz prethodnog dela imamo rastojanje a (od A do centra dimnjaka)
  - Pošto su mereni uglovi do dimnjaka, treba nam dužina a' (od A do dimnjaka)



Vežba 2

- Računanje visine dimnjaka



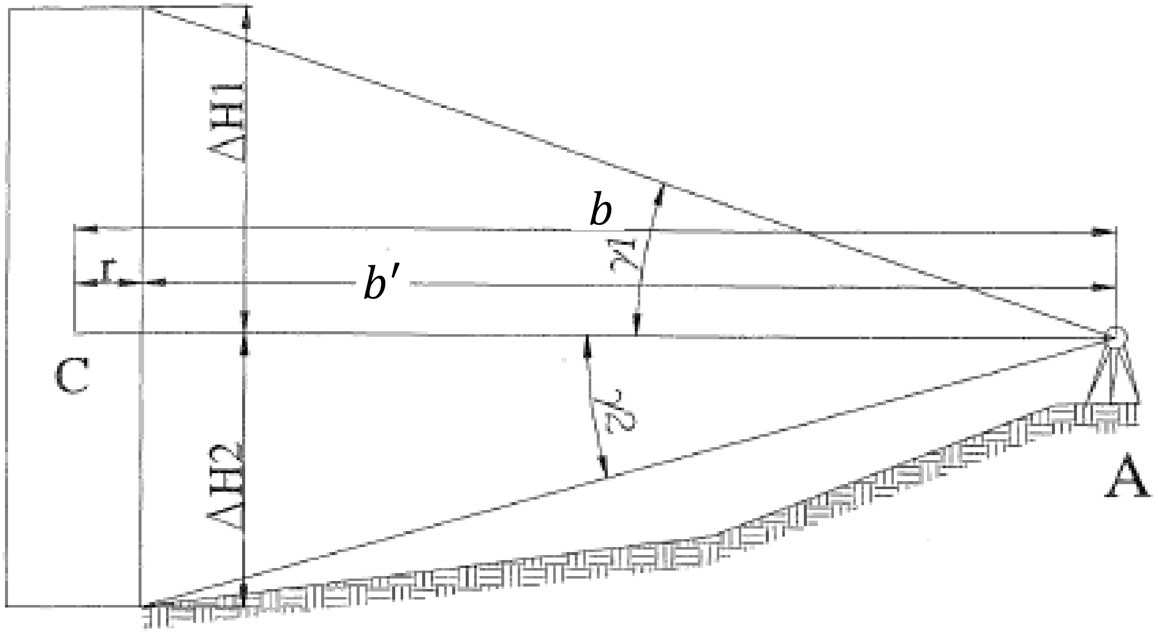
$b' = b - r =$

$H = \Delta H_1 + \Delta H_2 =$

$\Delta H_1 = b' \cdot tg\gamma_1$ $\Delta H_2 = b' \cdot tg\gamma_2$

Vežba 2

- Računanje visine dimnjaka



$$b' = b - r = 113.288m$$

$$H = \Delta H_1 + \Delta H_2 = 38.569m$$

$$\Delta H_1 = b' \cdot tg\gamma_1 \qquad \Delta H_2 = b' \cdot tg\gamma_2$$

Vežba 2

- Podsetnik iz statistike
  - Osobine varijanse
  - Varijansa konstante $Y = b$ jednaka je nuli $\sigma_b^2 = 0$
  - Varijansa zbira promenljive i konstante $Y = X + b$

    jednaka je varijansi promenljive $\sigma_{(X+b)}^2 = \sigma_X^2$
  - Varijansa proizvoda konstante i promenljive $Y = a \cdot X$

    jednaka je proizvodu kvadrata konstante i varijanse promenljive $\sigma_{(a \cdot X)}^2 = a^2 \cdot \sigma_X^2$
  - Varijansa zbira nezavisno promenljivih $Y = X_1 + X_2$

    jednaka je zbiru njihovih varijansi $\sigma_{(X_1+X_2)}^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2$

Vežba 2

- Opšti oblik funkcije nezavisnih merenja
  - Greške se prenose kroz merenja
  - Nelinearne funkcije se linearizuju razvojem u Tejlorov red

$$Y = f(l_1, l_2, \dots, l_n)$$

$$Y = f(l_1, l_2, \dots, l_n) + \frac{\partial f}{\partial l_1} dl_1 + \frac{\partial f}{\partial l_2} dl_2 + \dots + \frac{\partial f}{\partial l_n} dl_n$$

$$a_i = \frac{\partial f}{\partial l_i}; i = (1,2 \dots n)$$

$$Y = Y_0 + a_1 dl_1 + a_2 dl_2 + \dots + a_n dl_n$$

$$\sigma_Y^2 = \left(\frac{\partial f}{\partial l_1}\right)^2 \cdot \sigma_{l_1}^2 + \left(\frac{\partial f}{\partial l_2}\right)^2 \cdot \sigma_{l_2}^2 + \dots + \left(\frac{\partial f}{\partial l_n}\right)^2 \cdot \sigma_{l_n}^2$$

Vežba 2

- Računanje standardnih odstupanja funkcija merenja
  - Potrebne funkcije

$$d_{A-B} = \sqrt{(Y_B - Y_A)^2 + (X_B - X_A)^2}$$

$$a = \frac{d_{A-B}}{\sin\gamma}\sin\alpha$$

$$b = \frac{d_{A-B}}{\sin\gamma}\sin\beta$$

$$r = b \cdot \sin\delta_\alpha$$

$$r = a \cdot \sin\delta_\beta$$

$$H = b' \cdot tg\gamma_1 + b' \cdot tg\gamma_2$$

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Vežba 2

- Računanje standardnih odstupanja funkcija merenja

$$d_{A-B} = (y_B^2 - 2y_Ay_B + y_A^2 + x_B^2 - 2x_Ax_B + x_A^2)^{1/2'}$$

$$\partial d_{A-B} = \frac{1}{2}\overbrace{(y_B^2 - 2y_Ay_B + y_A^2 + x_B^2 - 2x_Ax_B + x_A^2)^{-1/2}}^{d_{A-B}}$$

$$*(y_B^2 - 2y_Ay_B + y_A^2 + x_B^2 - 2x_Ax_B + x_A^2)'$$

$$\partial d_{A-B} = \frac{1}{2}\frac{(y_B^2-2y_Ay_B+y_A^2+x_B^2-2x_Ax_B+x_A^2)'}{d_{A-B}}$$

Vežba 2

- Funkcija dužine između A i B

$$d_{A-B}=\sqrt{(Y_B-Y_A)^2+(X_B-X_A)^2}$$

$$\sigma_{d_{A-B}}=\sqrt{\left(\frac{\partial d_{A-B}}{\partial Y_A}\right)^2\cdot\sigma_{Y_A}^2+\left(\frac{\partial d_{A-B}}{\partial X_A}\right)^2\cdot\sigma_{X_A}^2+\left(\frac{\partial d_{A-B}}{\partial Y_B}\right)^2\cdot\sigma_{Y_B}^2+\left(\frac{\partial d_{A-B}}{\partial X_B}\right)^2\cdot\sigma_{X_B}^2}$$

$$\boxed{\frac{\partial d_{A-B}}{\partial Y_A}=-\frac{Y_B-Y_A}{d_{A-B}}}\quad\boxed{\frac{\partial d_{A-B}}{\partial Y_B}=\frac{Y_B-Y_A}{d_{A-B}}}\quad\boxed{\frac{\partial d_{A-B}}{\partial X_A}=-\frac{X_B-X_A}{d_{A-B}}}\quad\boxed{\frac{\partial d_{A-B}}{\partial X_B}=\frac{X_B-X_A}{d_{A-B}}}$$

Važi ako su standardi koordinata isti!!

$$\sigma_{d_{A-B}}=\sigma\sqrt{\frac{(-(Y_B-Y_A))^2}{d_{A-B}^2}+\frac{(Y_B-Y_A)^2}{d_{A-B}^2}+\frac{(-(X_B-X_A))^2}{d_{A-B}^2}+\frac{(X_B-X_A)^2}{d_{A-B}^2}}=$$

$$=\sigma\sqrt{\frac{2(Y_B-Y_A)^2+2(X_B-X_A)^2}{d_{A-B}^2}}=\sigma\sqrt{\frac{2\left[(Y_B-Y_A)^2+(X_B-X_A)^2\right]}{d_{A-B}^2}}=\sigma\sqrt{2}$$

Vežba 2

- Funkcije dužina a i b

$$a = \frac{d_{A-B}}{\sin\gamma}\sin\alpha \qquad b = \frac{d_{A-B}}{\sin\gamma}\sin\beta$$

$$\sigma_a = \sqrt{\left(\frac{\partial a}{\partial d_{A-B}}\right)^2 \cdot \sigma_{d_{A-B}}^2 + \left(\frac{\partial a}{\partial \gamma}\right)^2 \cdot \frac{\sigma_\gamma^2}{\rho^2} + \left(\frac{\partial a}{\partial \alpha}\right)^2 \cdot \frac{\sigma_\alpha^2}{\rho^2}} \qquad \sigma_b = \sqrt{\left(\frac{\partial b}{\partial d_{A-B}}\right)^2 \cdot \sigma_{d_{A-B}}^2 + \left(\frac{\partial b}{\partial \gamma}\right)^2 \cdot \frac{\sigma_\gamma^2}{\rho^2} + \left(\frac{\partial b}{\partial \beta}\right)^2 \cdot \frac{\sigma_\beta^2}{\rho^2}}$$

$$\frac{\partial a}{\partial d_{A-B}} = \frac{\sin\alpha}{\sin\gamma} \qquad \frac{\partial b}{\partial d_{A-B}} = \frac{\sin\beta}{\sin\gamma}$$

$$\frac{\partial a}{\partial \alpha} = \frac{d_{A-B}\cos\alpha}{\sin\gamma} \qquad \frac{\partial b}{\partial \beta} = \frac{d_{A-B}\cos\beta}{\sin\gamma}$$

$$\frac{\partial a}{\partial \gamma} = -\frac{d_{A-B}\sin\alpha\cos\gamma}{\sin^2\gamma} \qquad \frac{\partial b}{\partial \gamma} = -\frac{d_{A-B}\sin\beta\cos\gamma}{\sin^2\gamma}$$

Vežba 2

- Funkcije prečnika

$$r = b \cdot \sin \delta_\alpha$$

$$r = a \cdot \sin \delta_\beta$$

$$\delta_\alpha = \frac{\alpha_1 - \alpha_2}{2}$$

$$\sigma_{\delta_\alpha} = \sqrt{\left(\frac{\partial \delta_\alpha}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial \delta_\alpha}{\partial \alpha_2}\right)^2 \sigma_{\alpha_2}^2} = \sigma_u \sqrt{\left(\frac{\partial \delta_\alpha}{\partial \alpha_1}\right)^2 + \left(\frac{\partial \delta_\alpha}{\partial \alpha_2}\right)^2} = \frac{\sigma_u}{\sqrt{2}}$$

$$\sigma_r = \sqrt{\left(\frac{\partial r}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial r}{\partial \delta_\alpha}\right)^2 \frac{\sigma_{\delta_\alpha}^2}{\rho^2}}$$

Važi ako su standardi uglova isti!!

$$\frac{\partial r}{\partial a} = \sin \delta_\alpha$$

$$\frac{\partial r}{\partial \delta_\alpha} = a \cos \delta_\alpha$$

Preko dužine *b* odraditi samostalno!!!

Vežba 2

$$b' = b - r$$

- Funkcije visine

$$H = b' \tan\gamma_1 + b' \tan\gamma_2$$

$$\sigma_{b'} = \sqrt{\left(\frac{\partial b'}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial b'}{\partial r}\right)^2 \sigma_r^2}$$

$$\sigma_H = \sqrt{\left(\frac{\partial H}{\partial b'}\right)^2 \sigma_{b'}^2 + \left(\frac{\partial H}{\partial \gamma_1}\right)^2 \frac{\sigma_{\gamma_1}^2}{\rho^2} + \left(\frac{\partial H}{\partial \gamma_2}\right)^2 \frac{\sigma_{\gamma 2}^2}{\rho^2}} = \sqrt{\left(\frac{\partial H}{\partial b'}\right)^2 \sigma_{b'}^2 + \left[\left(\frac{\partial H}{\partial \gamma_1}\right)^2 + \left(\frac{\partial H}{\partial \gamma_2}\right)^2\right] \frac{\sigma_u^2}{\rho^2}}$$

$$\frac{\partial H}{\partial b'} = \tan\gamma_1 + \tan\gamma_2$$

$$\frac{\partial H}{\partial \gamma_1} = \frac{b'}{\cos^2\gamma_1}$$

$$\frac{\partial H}{\partial \gamma_2} = \frac{b'}{\cos^2\gamma_2}$$

Standard	Vrednost
$\sigma_{d_{A-B}}$	
σ_a	
σ_b	
σ_r	
σ_H	

Standard	Vrednost
$\sigma_{d_{A-B}}$	7,1 mm
σ_a	7,5 mm
σ_b	7,4 mm
σ_r	1,2 mm
σ_H	3,5 mm