



FAKULTET TEHNIČKIH NAUKA
DEPARTMAN ZA GRAĐEVINARSTVO I GEODEZIJU
LABORATORIJA ZA GEODEZIJU



INŽENJERSKA GEODEZIJA 1

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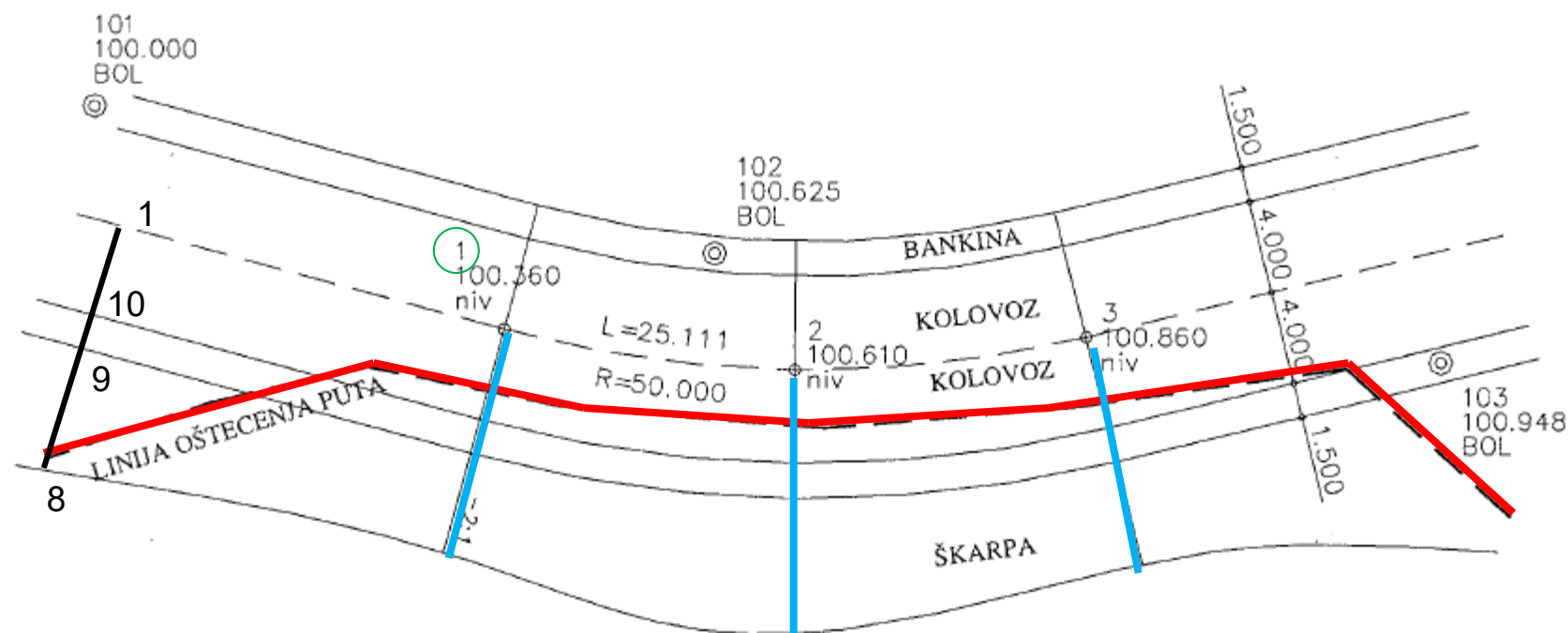
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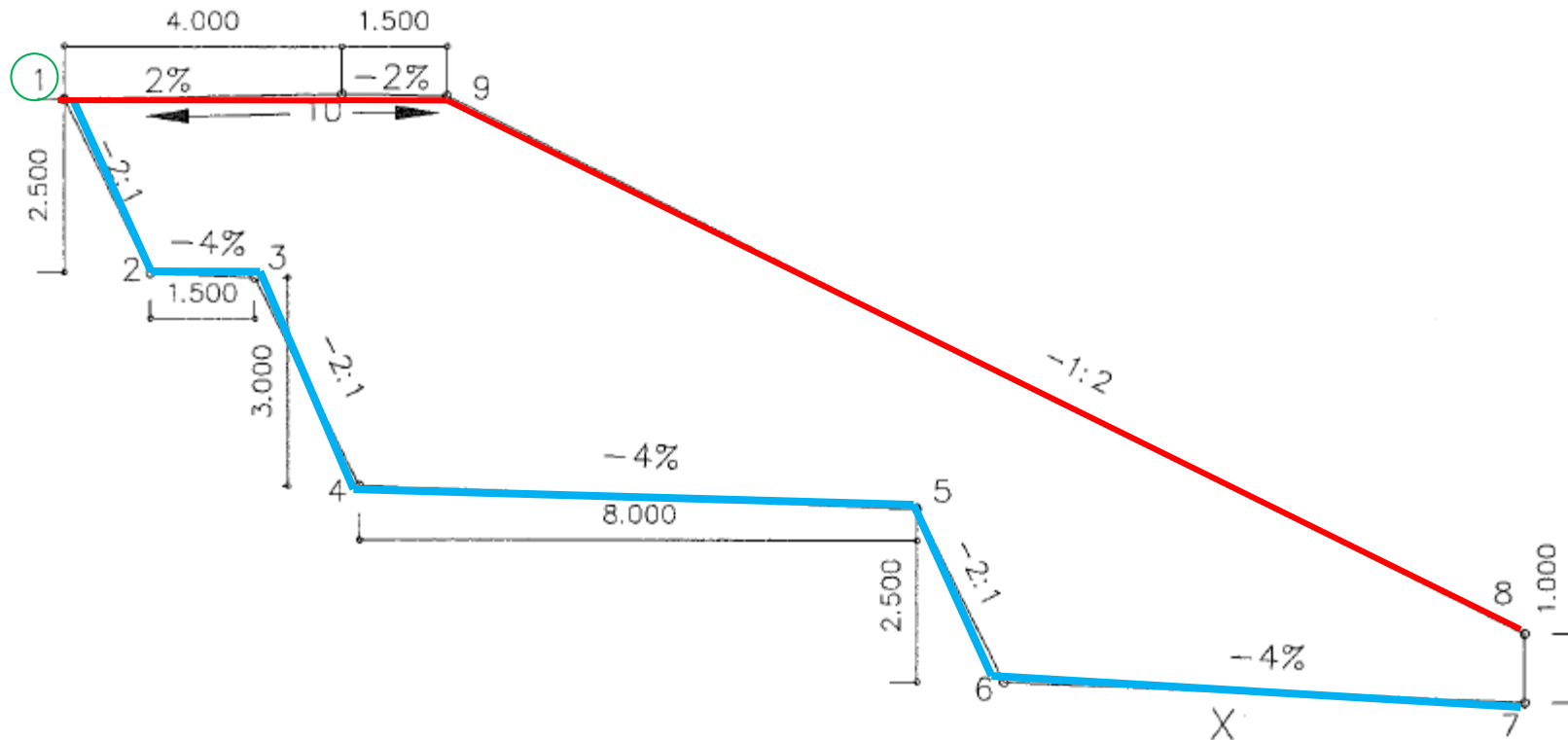


Vežba 5 Rekonstrukcija puta na nestabilnom terenu

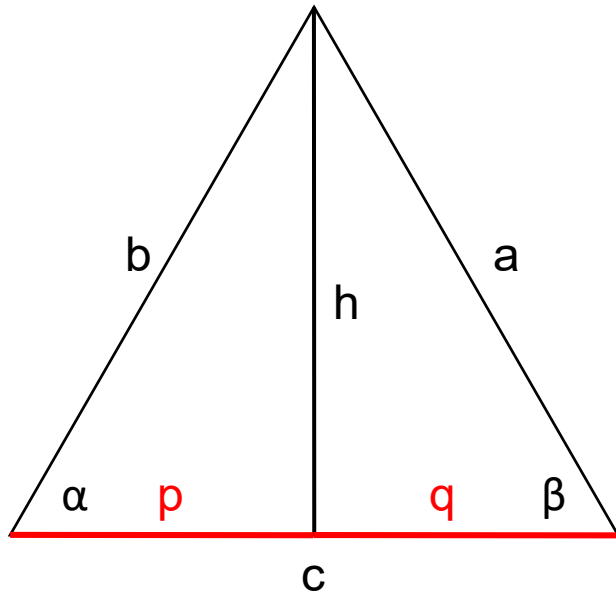
- Izračunati koordinate i kote tačaka na kaskadama
- Izračunati podatke za detaljno obeležavanje tačaka na kaskadama sa postojeće mreže
- Situacija:



Vežba 5



Vežba 5



$$p + q = c$$

$$\frac{p + q}{2} = \frac{c}{2}$$

$$h^2 = a^2 - q^2$$

$$h^2 = b^2 - p^2$$

$$b^2 - p^2 = a^2 - q^2$$

$$b^2 - a^2 = p^2 - q^2$$

$$b^2 - a^2 = (p - q)(p + q)$$

$$\frac{b^2 - a^2}{c} = p - q$$

$$\frac{p - q}{2} = \frac{b^2 - a^2}{2c}$$

$$p = \frac{p + q}{2} + \frac{p - q}{2}$$

$$q = \frac{p + q}{2} - \frac{p - q}{2}$$

$$\alpha = \arccos \frac{p}{b}$$

$$\beta = \arccos \frac{q}{a}$$

Vežba 5

❖ Lučni presek se primenjuje kada su u trouglu samo dužine poznate

$$v_1^3 = 90^\circ 37' 38''$$

$$\frac{p+q}{2} = \frac{d_{1-3}}{2} =$$

$$p = \frac{p+q}{2} + \frac{p-q}{2} =$$

$$q = \frac{p+q}{2} - \frac{p-q}{2} =$$

$$\alpha = \arccos \frac{p}{R} =$$

$$\beta = \arccos \frac{q}{R} =$$

$$d_{1-3} = 24.847m$$

$$R = a = b$$

$$\frac{p-q}{2} = \frac{b^2 - a^2}{2 * d_{1-3}} =$$

$$v_1^{ck} = v_1^3 - \alpha =$$

$$v_3^{ck} = v_3^1 + \beta =$$

$$y_{ck}' = y_1 + R * \sin v_1^{ck}$$

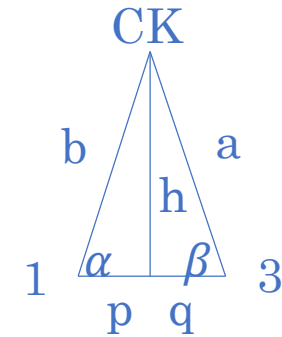
$$x_{ck}' = x_1 + R * \cos v_1^{ck}$$

$$y_{ck} =$$

$$y_{ck}'' = y_3 + R * \sin v_3^{ck}$$

$$x_{ck}'' = x_3 + R * \cos v_3^{ck}$$

$$x_{ck} =$$



usvojićemo $R = 50$



Vežba 5

❖ Lučni presek se primenjuje kada su u trouglu samo dužine poznate

$$v_1^3 = 90^\circ 37' 38''$$

$$d_{1-3} = 24.847m \quad R = a = b$$

$$\frac{p+q}{2} = \frac{d_{1-3}}{2} = 12.424m$$

$$\frac{p-q}{2} = \frac{b^2 - a^2}{2 * d_{1-3}} = 0$$

$$p = \frac{p+q}{2} + \frac{p-q}{2} = 12.424m$$

$$v_1^{ck} = v_1^3 - \alpha = 15^\circ 00' 52''$$

$$q = \frac{p+q}{2} - \frac{p-q}{2} = 12.424m$$

$$v_3^{ck} = v_3^1 + \beta = 346^\circ 14' 22''$$

$$\alpha = \arccos \frac{p}{R} = 75^\circ 36' 45''$$

$$y_{ck}' = y_1 + R * \sin v_1^{ck}$$

$$y_{ck}'' = y_3 + R * \sin v_3^{ck}$$

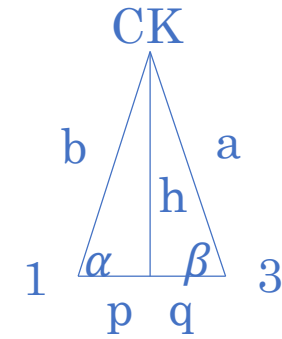
$$x_{ck}' = x_1 + R * \cos v_1^{ck}$$

$$x_{ck}'' = y_3 + R * \cos v_3^{ck}$$

$$\beta = \arccos \frac{q}{R} = 75^\circ 36' 45''$$

$$y_{ck} = 5030.461m$$

$$x_{ck} = 3038.741m$$



usvojićemo $R = 50m$



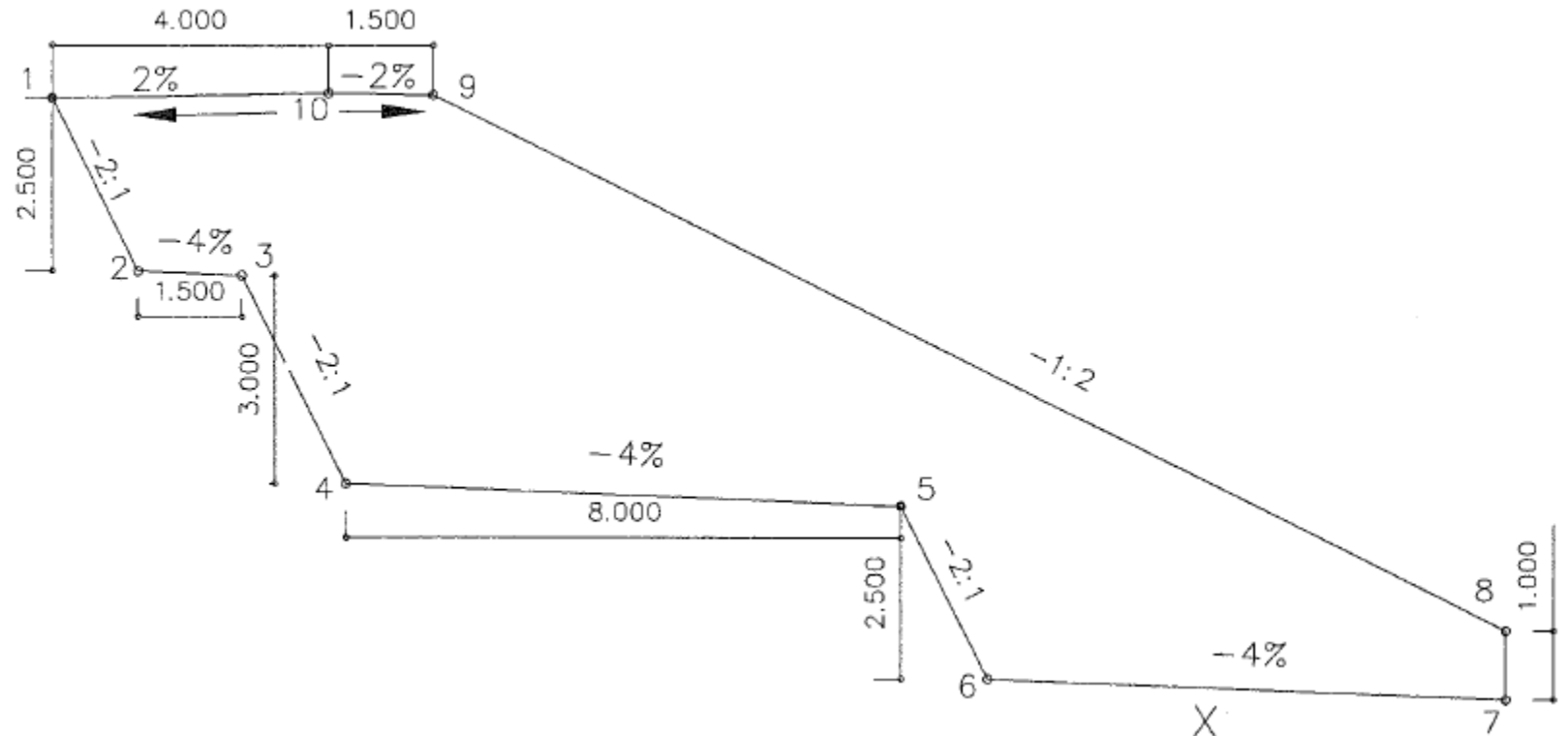
Vežba 5

Za računanje koordinata tačaka u profilima, usvojene su oznake:

1. profil (')
2. profil (")
3. profil (''')

1.1. a) Računanje kota tačaka na profilu 1

$$\begin{aligned}
 H_1 &= 100,360 \text{ m} \\
 H_2' &= H_1 - 2,500 = \\
 H_3' &= H_2' - \frac{1,500 \cdot 4}{100} = \\
 H_4' &= H_3' - 3,000 = \\
 H_5' &= H_4' - \frac{8,000 \cdot 4}{100} = \\
 H_6' &= H_5' - 2,500 = \\
 H_7' &= H_6' - \frac{x \cdot 4}{100} \\
 H_8' &= H_7' + 1,000 \\
 H_{10}' &= H_1 + \frac{4,000 \cdot 2}{100} =
 \end{aligned}$$





Vežba 5

Za računanje koordinata tačaka u profilima, usvojene su oznake:

1. profil (")
2. profil (")
3. profil (")

1.1. a) Računanje kota tačaka na profilu 1

$$H_1 = 100,360 \text{ m}$$

$$H_2 = H_1 - 2,500 = 97,860 \text{ m}$$

$$H_3 = H_2 - \frac{1,500 \cdot 4}{100} = 97,800 \text{ m}$$

$$H_4 = H_3 - 3,000 = 94,800 \text{ m}$$

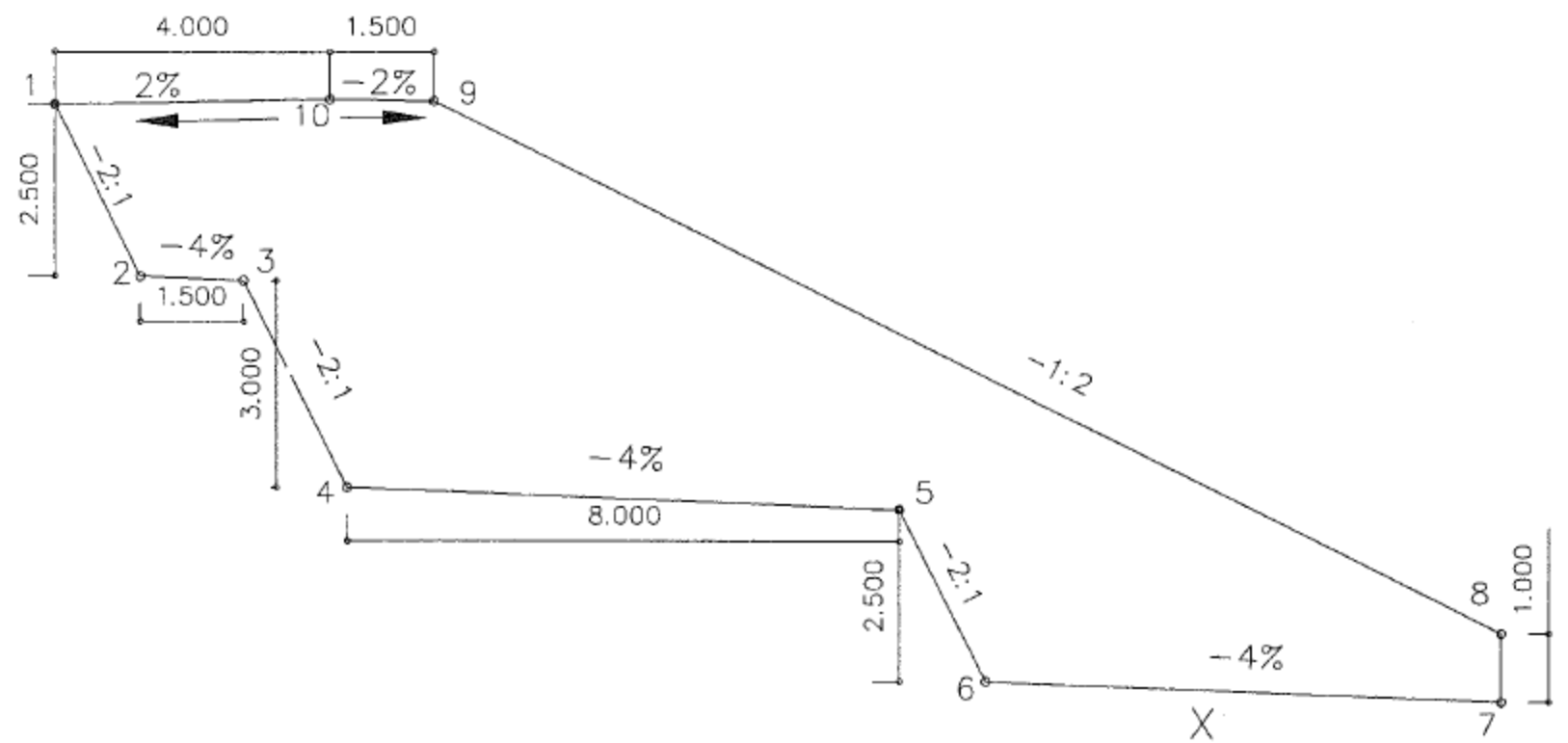
$$H_5 = H_4 - \frac{8,000 \cdot 4}{100} = 94,480 \text{ m}$$

$$H_6 = H_5 - 2,500 = 91,980 \text{ m}$$

$$H_7 = H_6 - \frac{x \cdot 4}{100}$$

$$H_8 = H_7 + 1,000$$

$$H_{10} = H_1 + \frac{4,000 \cdot 2}{100} = 100,440$$



Vežba 5

$$H'_9 = H'_{10} - \frac{1,500 \cdot 2}{100} =$$

Računanje nepoznate veličine x :

$$H'_7 = H'_9 - \frac{(1,25 + 1,50 + 1,50 + 8,00 + 1,25 + x - 4,00 - 1,50)}{2} \cdot 1 = H'_9 - \frac{8,000 + x}{2} \cdot 1$$

$$H'_7 = H'_6 - \frac{x \cdot 4}{100}$$

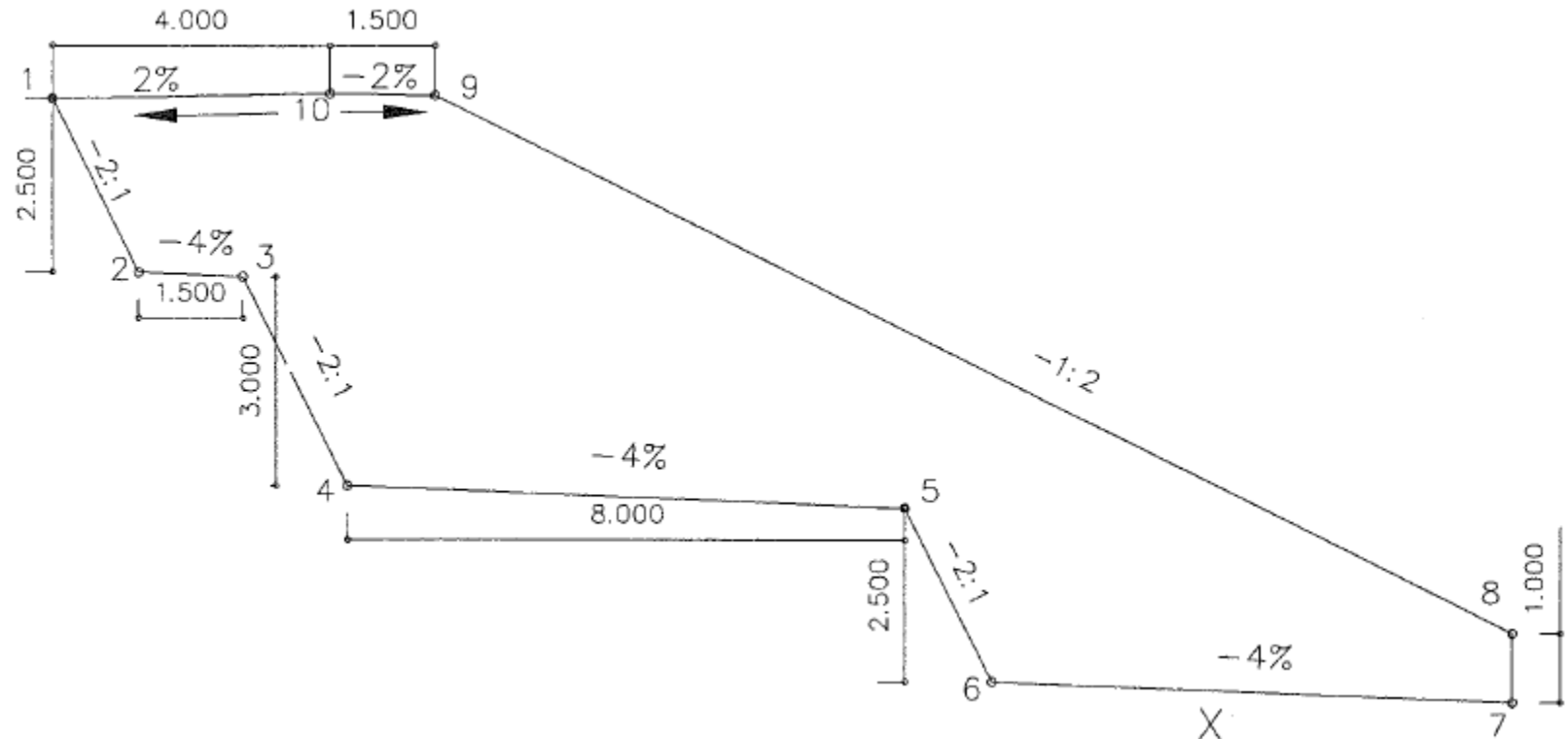
$\Rightarrow x =$

$$\overline{H}_7 =$$

$$H'_8 =$$

$$H'_9 =$$

$$H'_{10} =$$



Vežba 5

$$H'_9 = H'_{10} - \frac{1,500 \cdot 2}{100} =$$

Računanje nepoznate veličine x:

$$H'_7 = H'_9 - \frac{(1,25 + 1,50 + 1,50 + 8,00 + 1,25 + x - 4,00 - 1,50)}{2} \cdot 1 = H'_9 - \frac{8,000 + x}{2} \cdot 1 \Rightarrow$$

$$H'_7 = H'_6 - \frac{x \cdot 4}{100}$$

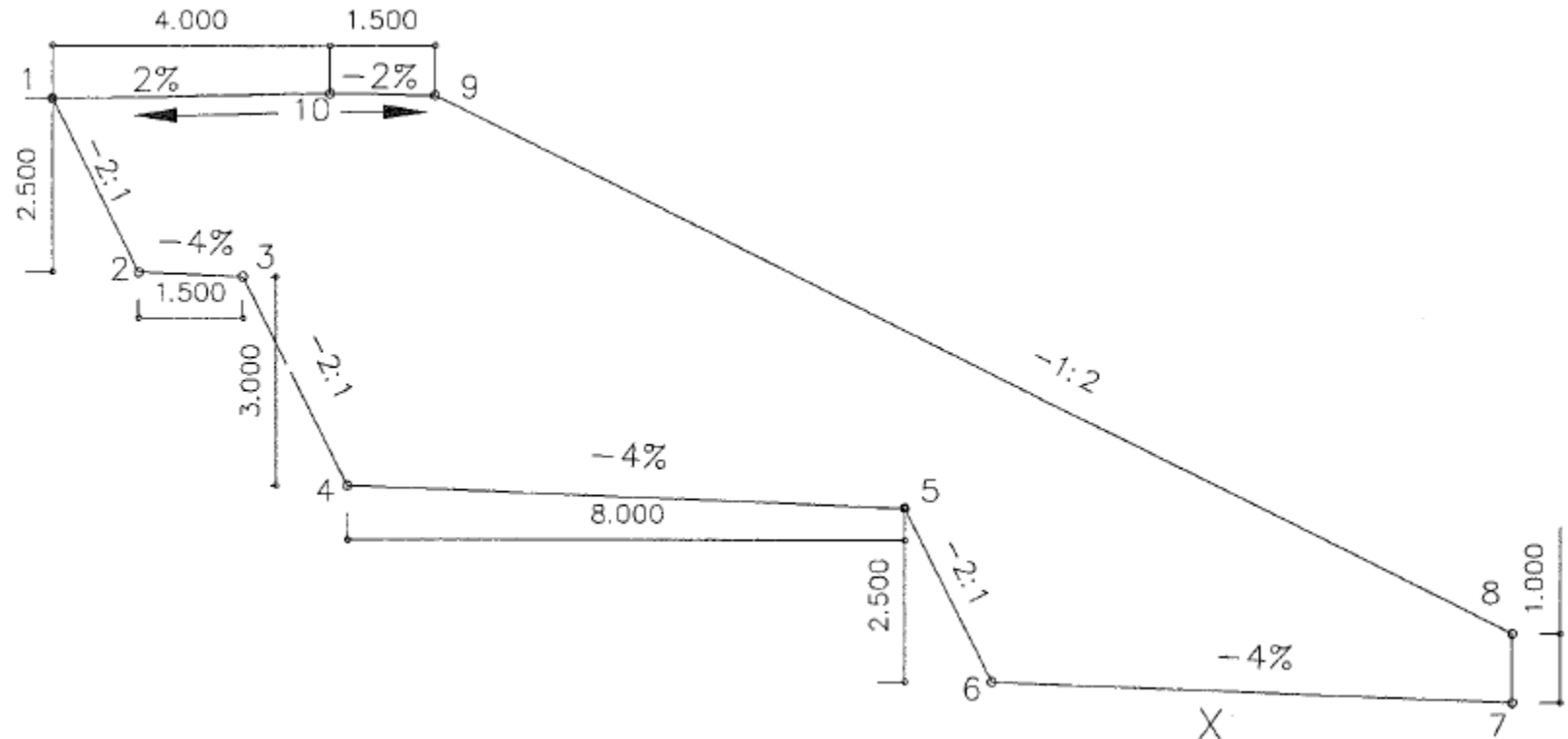
$$\Rightarrow x = \frac{100 \cdot H'_9 - 500 - 100 \cdot H'_6}{46} = 7,457 \text{ m}$$

$$H'_7 = 91,682 \text{ m}$$

$$H'_8 = 92,682 \text{ m}$$

$$H'_9 = 100,410 \text{ m}$$

$$H'_{10} = 100,440 \text{ m}$$



Vežba 5

1.1. b) Računanje koordinata tačaka na profilu 1

$$v_{C1}^1 = \arctg\left(\frac{Y_1 - Y_{C1}}{X_1 - X_{C1}}\right)$$

$$Y_2' = Y_1 + 1,250 \cdot \sin v_{C1}^1$$

$$X_2' = X_1 + 1,250 \cdot \cos v_{C1}^1$$

$$Y_3' = Y_1 + (1,250 + 1,500) \cdot \sin v_{C1}^1$$

$$X_3' = X_1 + (1,250 + 1,500) \cdot \cos v_{C1}^1$$

$$Y_4' = Y_1 + (1,250 + 1,500 + 1,500) \cdot \sin v_{C1}^1$$

$$X_4' = X_1 + (1,250 + 1,500 + 1,500) \cdot \cos v_{C1}^1$$

$$Y_5' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \sin v_{C1}^1$$

$$X_5' = X_1 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \cos v_{C1}^1$$

$$Y_6' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \sin v_{C1}^1$$

$$X_6' = X_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \cos v_{C1}^1$$

$$Y_7' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \sin v_{C1}^1$$

$$X_7' = X_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \cos v_{C1}^1$$

$$Y_8' = Y_7' ; X_8' = X_7'$$

$$Y_9' = Y_1 + (4,000 + 1,500) \cdot \sin v_{C1}^1$$

$$X_9' = X_1 + (4,000 + 1,500) \cdot \cos v_{C1}^1$$

$$Y_{10}' = Y_1 + 4,000 \cdot \sin v_{C1}^1$$

$$X_{10}' = X_1 + 4,000 \cdot \cos v_{C1}^1$$



Vežba 5

$$\nu_{C1}^1 = \arctg\left(\frac{Y_1 - Y_{C1}}{X_1 - X_{C1}}\right) = 195^\circ 00' 52''$$

$$Y_2' = Y_1 + 1,250 \cdot \sin \nu_{C1}^1 = 5017,184 \text{ m}$$

$$X_2' = X_1 + 1,250 \cdot \cos \nu_{C1}^1 = 2989,241 \text{ m}$$

$$Y_3' = Y_1 + (1,250 + 1,500) \cdot \sin \nu_{C1}^1 = 5016,796 \text{ m}$$

$$X_3' = X_1 + (1,250 + 1,500) \cdot \cos \nu_{C1}^1 = 2987,792 \text{ m}$$

$$Y_4' = Y_1 + (1,250 + 1,500 + 1,500) \cdot \sin \nu_{C1}^1 = 5016,407 \text{ m}$$

$$X_4' = X_1 + (1,250 + 1,500 + 1,500) \cdot \cos \nu_{C1}^1 = 2986,343 \text{ m}$$

$$Y_5' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \sin \nu_{C1}^1 = 5014,335 \text{ m}$$

$$X_5' = X_1 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \cos \nu_{C1}^1 = 2978,616 \text{ m}$$

$$Y_6' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \sin \nu_{C1}^1 = 5014,011 \text{ m}$$

$$X_6' = X_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \cos \nu_{C1}^1 = 2977,409 \text{ m}$$

$$Y_7' = Y_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \sin \nu_{C1}^1 = 5012,079 \text{ m}$$

$$X_7' = X_1 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \cos \nu_{C1}^1 = 2970,206 \text{ m}$$

$$Y_8' = Y_7' ; X_8' = X_7'$$

$$Y_9' = Y_1 + (4,000 + 1,500) \cdot \sin \nu_{C1}^1 = 5016,083 \text{ m}$$

$$X_9' = X_1 + (4,000 + 1,500) \cdot \cos \nu_{C1}^1 = 2985,136 \text{ m}$$

$$Y_{10}' = Y_1 + 4,000 \cdot \sin \nu_{C1}^1 = 5016,472 \text{ m}$$

$$X_{10}' = X_1 + 4,000 \cdot \cos \nu_{C1}^1 = 2986,585 \text{ m}$$



Vežba 5

1. 2. a) Računanje kota tačaka na profilu 2

Kote tačaka u profilu 2 se računaju na isti način kao i u profilu 1.

$$x = 7,457\text{m}$$

Kote su:

$$H_1 = 100,610\text{ m} \quad (\text{kota tačke na osovini puta u profilu 2})$$

Ostale kote su:

$$H_2'' = \quad ; \quad H_3'' = \quad ; \quad H_4'' =$$

$$H_5'' = \quad ; \quad H_6'' = \quad ; \quad H_7'' =$$

$$H_8'' = \quad ; \quad H_9'' = \quad ; \quad H_{10}'' =$$

1.2. b) Računanje koordinata tačaka na profilu 2

$$v_{C1}'' = \arctg\left(\frac{Y_2 - Y_{C1}}{X_2 - X_{C1}}\right) =$$

$$Y_2'' = Y_2 + 1,250 \cdot \sin v_{C1}'' =$$

$$X_2'' = X_2 + 1,250 \cdot \cos v_{C1}'' =$$

$$Y_3'' = Y_2 + (1,250 + 1,500) \cdot \sin v_{C1}'' =$$

$$X_3'' = X_2 + (1,250 + 1,500) \cdot \cos v_{C1}'' =$$

$$Y_4'' = Y_2 + (1,250 + 1,500 + 1,500) \cdot \sin v_{C1}'' =$$

$$X_4'' = X_2 + (1,250 + 1,500 + 1,500) \cdot \cos v_{C1}'' =$$

$$Y_5'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \sin v_{C1}'' =$$

$$X_5'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \cos v_{C1}'' =$$

$$Y_6'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \sin v_{C1}'' =$$

$$X_6'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \cos v_{C1}'' =$$

$$Y_7'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \sin v_{C1}'' =$$

$$X_7'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \cos v_{C1}'' =$$

$$Y_8'' = Y_7'' \quad ; \quad X_8'' = X_7''$$

$$Y_9'' = Y_2 + (4,000 + 1,500) \cdot \sin v_{C1}'' =$$

$$X_9'' = X_2 + (4,000 + 1,500) \cdot \cos v_{C1}'' =$$

$$Y_{10}'' = Y_2 + 4,000 \cdot \sin v_{C1}'' =$$

$$X_{10}'' = X_2 + 4,000 \cdot \cos v_{C1}'' =$$

3I ISTI KAO DRUGI
SAMO SE KORISTI TACKA 3



Vežba 5

1. 2. a) Računanje kota tačaka na profilu 2

Kote tačaka u profilu 2 se računaju na isti način kao i u profilu 1.

$$x = 7,457 \text{ m}$$

Kote su:

$$H_1 = 100,610 \text{ m} \quad (\text{kota tačke na osovini puta u profilu 2})$$

Ostale kote su:

$$H_2'' = 98,110 \text{ m} \quad ; \quad H_3'' = 98,050 \text{ m} \quad ; \quad H_4'' = 95,050 \text{ m}$$

$$H_5'' = 94,730 \text{ m} \quad ; \quad H_6'' = 92,230 \text{ m} \quad ; \quad H_7'' = 91,932 \text{ m}$$

$$H_8'' = 92,932 \text{ m} \quad ; \quad H_9'' = 100,660 \text{ m} \quad ; \quad H_{10}'' = 100,690 \text{ m}$$

1.2. b) Računanje koordinata tačaka na profilu 2

$$v_{C1}'' = \arctg\left(\frac{Y_2 - Y_{C1}}{X_2 - X_{C1}}\right) = 180^\circ 37' 36''$$

$$Y_2'' = Y_2 + 1,250 \cdot \sin v_{C1}'' = 5029,900 \text{ m}$$

$$X_2'' = X_2 + 1,250 \cdot \cos v_{C1}'' = 2987,494 \text{ m}$$

$$Y_3'' = Y_2 + (1,250 + 1,500) \cdot \sin v_{C1}'' = 5029,884 \text{ m}$$

$$X_3'' = X_2 + (1,250 + 1,500) \cdot \cos v_{C1}'' = 2985,994 \text{ m}$$

$$Y_4'' = Y_2 + (1,250 + 1,500 + 1,500) \cdot \sin v_{C1}'' = 5029,868 \text{ m}$$

$$X_4'' = X_2 + (1,250 + 1,500 + 1,500) \cdot \cos v_{C1}'' = 2984,494 \text{ m}$$

$$Y_5'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \sin v_{C1}'' = 5029,777 \text{ m}$$

$$X_5'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000) \cdot \cos v_{C1}'' = 2976,495 \text{ m}$$

$$Y_6'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \sin v_{C1}'' = 5029,766 \text{ m}$$

$$X_6'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250) \cdot \cos v_{C1}'' = 2975,245 \text{ m}$$

$$Y_7'' = Y_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \sin v_{C1}'' = 5029,685 \text{ m}$$

$$X_7'' = X_2 + (1,250 + 1,500 + 1,500 + 8,000 + 1,250 + x) \cdot \cos v_{C1}'' = 2967,788 \text{ m}$$

$$Y_8'' = Y_7'' \quad ; \quad X_8'' = X_7''$$

$$Y_9'' = Y_2 + (4,000 + 1,500) \cdot \sin v_{C1}'' = 5029,845 \text{ m}$$

$$X_9'' = X_2 + (4,000 + 1,500) \cdot \cos v_{C1}'' = 2983,244 \text{ m}$$

$$Y_{10}'' = Y_2 + 4,000 \cdot \sin v_{C1}'' = 5029,870 \text{ m}$$

$$X_{10}'' = X_2 + 4,000 \cdot \cos v_{C1}'' = 2984,744 \text{ m}$$

3I ISTI KAO DRUGI
SAMO SE KORISTI TACKA 3



Vežba 5

2) Računanje podataka za detaljno obeležavanje tačaka na kaskadama

Stanica	Vizura	Y [m]	X [m]	v	Dužina
102		5026,453	2993,725		
	101	5000,000	3000,000		
	103	5057,439	2989,073		
Profil 1					
	6'	5014,011	2977,409		
	5'	5014,335	2978,616		
	9'	5016,083	2985,136		
	4'	5016,407	2986,343		
	10'	5016,472	2986,585		
	3'	5016,796	2987,792		
	2'	5017,184	2989,241		
	1'	5017,508	2990,448		
	7'	5012,079	2970,206		
	8'	5012,079	2970,206		



Vežba 5

2) Računanje podataka za detaljno obeležavanje tačaka na kaskadama

Stanica	Vizura	Y [m]	X [m]	v	Dužina
102		5026,453	2993,725		
	101	5000,000	3000,000	283°20'40"	27,187
	103	5057,439	2989,073	98°32'17"	31,333
Profil 1					
	6'	5014,011	2977,409	217°19'40"	20,519
	5'	5014,335	2978,616	218°43'51"	19,368
	9'	5016,083	2985,136	230°21'59"	13,465
	4'	5016,407	2986,343	233°41'26"	12,467
	10'	5016,472	2986,585	234°25'17"	12,272
	3'	5016,796	2987,792	238°26'04"	11,334
	2'	5017,184	2989,241	244°11'02"	10,297
	1'	5017,508	2990,448	249°52'46"	9,526
	7'	5012,079	2970,206	211°25'54"	27,564
	8'	5012,079	2970,206	211°25'54"	27,564