FAKULTET TEHNIČKIH NAUKA
GEODEZIJA I GEOINFORMATIKA

RAČUN IZRAVNANJA
- VEŽBA 5 -

NOVI SAD, 2025.

Izravnanje po metodi posrednih merenja

- Gaus-Markovljev model izravnanja

$\mathbf{v} = \mathbf{A}\mathbf{x} + \mathbf{f}$ – funkcionalni model

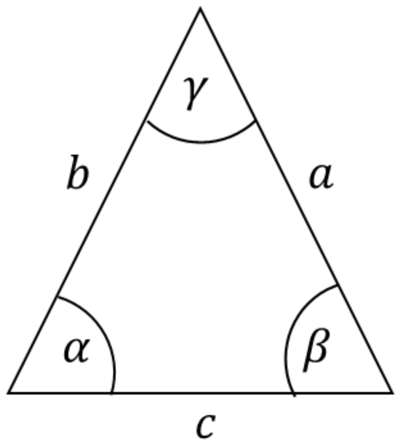
$\mathbf{P}_l = \mathbf{Q}_l^{-1}, E(\mathbf{v}) = \mathbf{0}$ – stohastički model

- Funkcionalnim modelom je definisana funkcionalna (matematička) veza između merenih veličina i nepoznatih parametara modela.
- Stohastički model definiše određene pretpostavke u vezi sa merenjima.

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

U trouglu su mereni uglovi i dužine: α, β, γ, a, b i c. Standard merenja pravaca iznosi 2”, a dužina 3mm + 3mm/km.
Izravnati rezultate merenja i oceniti tačnost merenja i izravnatih veličina.



Funkcije veze:

$$\hat{\alpha} = \alpha + v_\alpha = X$$

$$\hat{\beta} = \beta + v_\beta = Y$$

$$\hat{\gamma} = \gamma + v_\gamma = 180° - (X + Y)$$

$$\hat{a} = a + v_a = Z$$

$$\hat{b} = b + v_b = \frac{Z}{\sin(X)} \cdot \sin(Y)$$

$$\hat{c} = c + v_c = \frac{Z}{\sin(X)} \cdot \sin\big(180° - (X + Y)\big)$$

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

  Neke od ovih funkcija linearizuju se razvojem u Tejlorov red u okolini približnih vrednosti nepoznatih parametara, nakon čega se dobijaju jednačine popravaka:

  $v_\alpha = dX + f_\alpha, \;\; f_\alpha = X_0 - \alpha$

  $v_\beta = dY + f_\beta, \;\; f_\beta = Y_0 - \beta$

  $v_\gamma = -dX - dY + f_\gamma, \;\; f_\gamma = \left(180° - (X_0 + Y_0)\right) - \gamma$

  $v_a = dZ + f_a, \;\; f_a = Z_0 - a$

  $v_b = a_1 dX + b_1 dY + c_1 dZ + f_b, \;\; f_b = \left(\frac{Z_0}{\sin(X_0)} \cdot \sin(Y_0)\right) - b$

  $v_c = a_2 dX + b_2 dY + c_2 dZ + f_c, \;\; f_c = \left(\frac{Z_0}{\sin(X_0)} \cdot \sin\left(180° - (X_0 + Y_0)\right)\right) - c$

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

  Koeficijenti a_i, b_i i c_i formiraju se na sledeći način:

$$a_1 = \left(\frac{\partial b}{\partial X}\right)_0 = \frac{-Z_0 \cdot \sin Y_0 \cdot \cos X_0}{\sin^2 X_0} \cdot \frac{1}{\rho}$$

$$b_1 = \left(\frac{\partial b}{\partial Y}\right)_0 = \frac{Z_0 \cdot \cos Y_0}{\sin X_0} \cdot \frac{1}{\rho}$$

$$c_1 = \left(\frac{\partial b}{\partial Z}\right)_0 = \frac{\sin Y_0}{\sin X_0}$$

$$a_2 = \left(\frac{\partial c}{\partial X}\right)_0 = \frac{-Z_0 \cdot \cos[180° - (X_0 + Y_0)] \cdot \sin X_0 - Z_0 \cdot \sin[180° - (X_0 + Y_0)] \cdot \cos X_0}{\sin^2 X_0} \cdot \frac{1}{\rho}$$

$$b_2 = \left(\frac{\partial c}{\partial Y}\right)_0 = \frac{Z_0}{\sin X_0} \cdot (-\cos[180° - (X_0 + Y_0)]) \cdot \frac{1}{\rho}$$

$$c_2 = \left(\frac{\partial c}{\partial Z}\right)_0 = \frac{\sin(X_0 + Y_0)}{\sin(X_0)}$$

Funkcionalni model

- **Primer 1 – dvodimenzionalna geodetska mreža**

  Formiranje matrice dizajna $\mathbf{A}$ i vektora slobodnih članova $\mathbf{f}$:

$$\mathbf{A} = \begin{array}{c} \begin{matrix} dX & dY & dZ \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{matrix} \alpha \\ \beta \\ \gamma \\ a \\ b \\ c \end{matrix} \end{array} \qquad \mathbf{f} = \begin{bmatrix} f_\alpha \\ f_\beta \\ f_\gamma \\ f_a \\ f_b \\ f_c \end{bmatrix}$$

  Napomene:
  - Slobodna članove za horizontalne pravce izraziti u sekundama.
  - Slobodne članove za dužine izraziti u milimetrima.
  - Pri računanju koeficijenata a_i, b_i i c_i dužine Z_0 izraziti u milimetrima.

Stohastički model

- **Primer 1 – dvodimenzionalna geodetska mreža**

Formiranje matrice težina $\mathbf{P}$:

Težine merenja P_i predstavljaju stepen poverenja u rezultate merenja.

$\sigma_p = 2" \;\; \Rightarrow \;\; \sigma_\alpha = \sigma_\beta = \sigma_\gamma = \sqrt{2} \cdot 2“$

Za σ_0 usvojiti vrednost **1**!

$$\sigma_a = 3\,\text{mm} + 3\,\frac{\text{mm}}{\text{km}} \cdot a[\text{km}]$$

$$\sigma_b = 3\,\text{mm} + 3\,\frac{\text{mm}}{\text{km}} \cdot b[\text{km}]$$

$$\sigma_c = 3\,\text{mm} + 3\,\frac{\text{mm}}{\text{km}} \cdot c[\text{km}]$$

$$P_\alpha = \frac{\sigma_0^2}{\sigma_{\sigma_\alpha}^2}, P_a = \frac{\sigma_0^2}{\sigma_a^2}$$

$$P_\beta = \frac{\sigma_0^2}{\sigma_{\sigma_\beta}^2}, P_b = \frac{\sigma_0^2}{\sigma_b^2}$$

$$P_\gamma = \frac{\sigma_0^2}{\sigma_{\sigma_\gamma}^2}, P_c = \frac{\sigma_0^2}{\sigma_c^2}$$

$$\mathbf{P} = \begin{bmatrix} P_\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & P_\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & P_\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & P_a & 0 & 0 \\ 0 & 0 & 0 & 0 & P_b & 0 \\ 0 & 0 & 0 & 0 & 0 & P_c \end{bmatrix} \begin{matrix} \alpha \\ \beta \\ \gamma \\ a \\ b \\ c \end{matrix}$$

Kompletno rešenje primera 1 dostupno je u fajlu *Račun izravnanja - Vežba 5.xlsx*.

Primena metoda najmanjih kvadrata (MNK)

- Sistem normalnih jednačina

$$\mathbf{N}\hat{\mathbf{x}} + \mathbf{n} = \mathbf{0}$$

$$\mathbf{N} = \mathbf{A}^T\mathbf{P}\mathbf{A}$$

$$\mathbf{n} = \mathbf{A}^T\mathbf{P}\mathbf{f}$$

- Ocena nepoznatih parametara i popravaka merenih veličina

$$\hat{\mathbf{x}} = -\mathbf{Q}_{\hat{\mathbf{x}}} \cdot \mathbf{n},\ \mathbf{Q}_{\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

$$\hat{\mathbf{v}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}$$

Kontrola računanja:
$\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}} \equiv \mathbf{f}^T\mathbf{P}\mathbf{f} + \mathbf{n}^T\hat{\mathbf{x}}$

- Ocena disperzionog koeficijenta

$$m_0^2 = \frac{\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}}{f}$$

$f = n - u,\ n$ – broj merenja, u – broj nepoznatih parametara

Globlani test na grube greške

- Hipoteze

$H_0\colon \sigma^2 = \sigma_0^2$ protiv $H_a\colon \sigma^2 \neq \sigma_0^2$,

pri čemu je $\sigma^2 = M(m_0^2)$, a M operator matematičkog očekivanja.

- Test statistika

$$T = \frac{m_0^2}{\sigma_0^2} \sim F_{1-\alpha,f,\infty}$$

Excel: $F_{1-\alpha,f,\infty}$ $\rightarrow$ **FINV(α, f,10000)**

Ukoliko je $T < F_{1-\alpha,f,\infty}$, nulta hipoteza se ne odbacuje, tj. nema grubih grešaka.

Ukoliko je $T \geq F_{1-\alpha,f,\infty}$, nulta hipoteza se odbacuje, pa konstatujemo da su u merenjima prisutne grube greške.

Izravnate vrednosti merenja i nepoznatih parametara

- Izravnate vrednosti nepoznatih parametara

$$\hat{X} = X_0 + \widehat{dX}$$
$$\hat{Y} = Y_0 + \widehat{dY}$$
$$\hat{Z} = Z_0 + \widehat{dZ}$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \widehat{dX} \\ \widehat{dY} \\ \widehat{dZ} \end{bmatrix}$$

- Izravnate vrednosti merenih veličina

$$\hat{\alpha} = \alpha + \hat{v}_\alpha$$
$$\hat{\beta} = \beta + \hat{v}_\beta$$
$$\hat{\gamma} = \gamma + \hat{v}_\gamma$$
$$\hat{a} = a + \hat{v}_a$$
$$\hat{b} = b + \hat{v}_b$$
$$\hat{c} = c + \hat{v}_c$$

$$\hat{\mathbf{v}} = \begin{bmatrix} \hat{v}_\alpha \\ \hat{v}_\beta \\ \hat{v}_\gamma \\ \hat{v}_a \\ \hat{v}_b \\ \hat{v}_c \end{bmatrix}$$

Analiza tačnosti geodetskih mreža

- *A posteriori* standardna devijacija (globalna mera tačnosti)

$$m_0 = \sqrt{\frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f}},$$

pri čemu je $f = n - u$ broj stepeni slobode.

- Standardne devijacije nepoznatih parametara (lokalne mere tačnosti)

$$\hat{\sigma}_X = m_0 \sqrt{Q_{\hat{X}\hat{X}}},$$

$$\hat{\sigma}_Y = m_0 \sqrt{Q_{\hat{Y}\hat{Y}}},$$

$$\hat{\sigma}_Z = m_0 \sqrt{Q_{\hat{Z}\hat{Z}}},$$

$$\mathbf{Q}_{\hat{\mathbf{x}}} = \begin{bmatrix} Q_{\hat{X}\hat{X}} & Q_{\hat{X}\hat{Y}} & Q_{\hat{X}\hat{Z}} \\ Q_{\hat{Y}\hat{X}} & Q_{\hat{Y}\hat{Y}} & Q_{\hat{Y}\hat{Z}} \\ Q_{\hat{Z}\hat{X}} & Q_{\hat{Z}\hat{Y}} & Q_{\hat{Z}\hat{Z}} \end{bmatrix}$$

pri čemu su $Q_{\hat{X}\hat{X}}$, $Q_{\hat{Y}\hat{Y}}$ i $Q_{\hat{Z}\hat{Z}}$ dijagonalni elementi kofaktorske matrice $\mathbf{Q}_{\hat{\mathbf{x}}}$.

Funkcionalni model

- Jednačine popravaka – visinske razlike (geometrijski nivelman)

$$\widehat{\Delta h}_{i-j} = \Delta h_{i-j} + v_{\Delta h_{i-j}} \qquad (1)$$

$$\widehat{\Delta h}_{i-j} = \widehat{H}_j - \widehat{H}_i \qquad (2)$$

Na osnovu izraza (1) i (2) može se napisati:

$$v_{\Delta h_{i-j}} = \left(\widehat{H}_j - \widehat{H}_i\right) - \Delta h_{i-j},\ \widehat{H}_j = H_j^0 + dH_j \text{ i } \widehat{H}_i = H_i^0 + dH_i,$$

$$v_{\Delta h_{i-j}} = H_j^0 + dH_j - H_i^0 - dH_i - \Delta h_{i-j},$$

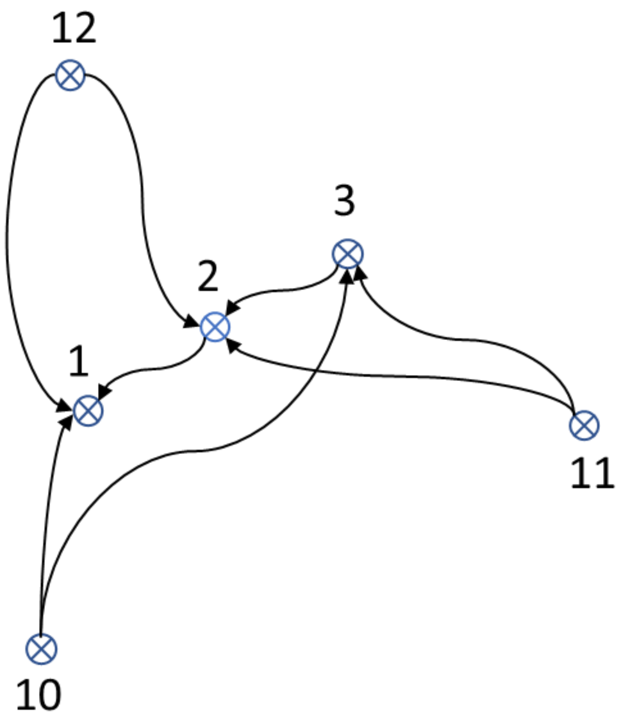
a onda:

$$v_{\Delta h_{i-j}} = dH_j - dH_i + f_{\Delta h_{i-j}},\ \ f_{\Delta h_{i-j}} = \left(H_j^0 - H_i^0\right) - \Delta h_{i-j}.$$

Funkcionalni model

- **Primer 2 – jednodimenzionalna geodetska mreža**

U nivelmanskoj mreži koja se sastoji od 6 repera merene su visinske razlike metodom geometrijskog nivelmana. Izravnati rezultate merenja i oceniti tačnost nepoznatih parametara.
Dati reperi: 10, 11 i 12
Nepoznati reperi: 1, 2 i 3



Približne visine repera:

$$H_{1,1}^0 = H_{10} + \Delta h_{10-1}, H_{1,2}^0 = H_{12} + \Delta h_{12-1}, H_1^0 = \frac{H_{1,1}^0 + H_{1,2}^0}{2}$$

$$H_{2,1}^0 = H_{11} + \Delta h_{11-2}, H_{2,2}^0 = H_{12} + \Delta h_{12-2}, H_2^0 = \frac{H_{2,1}^0 + H_{2,2}^0}{2}$$

$$H_{3,1}^0 = H_{10} + \Delta h_{10-3}, H_{3,2}^0 = H_{11} + \Delta h_{11-3}, H_3^0 = \frac{H_{3,1}^0 + H_{3,2}^0}{2}$$

Funkcionalni model

- **Primer 2 – jednodimenzionalna geodetska mreža**

  Jednačine popravaka:

  $v_{\Delta h_{10-1}} = dH_1 + f_{\Delta h_{10-1}}, \;\; f_{\Delta h_{10-1}} = (H_1^0 - H_{10}) - \Delta h_{10-1}$

  $v_{\Delta h_{10-3}} = dH_3 + f_{\Delta h_{10-3}}, \;\; f_{\Delta h_{10-3}} = (H_3^0 - H_{10}) - \Delta h_{10-3}$

  $v_{\Delta h_{11-2}} = dH_2 + f_{\Delta h_{11-2}}, \;\; f_{\Delta h_{11-2}} = (H_2^0 - H_{11}) - \Delta h_{11-2}$

  $v_{\Delta h_{11-3}} = dH_3 + f_{\Delta h_{11-3}}, \;\; f_{\Delta h_{11-3}} = (H_3^0 - H_{11}) - \Delta h_{11-3}$

  $v_{\Delta h_{12-1}} = dH_1 + f_{\Delta h_{12-1}}, \;\; f_{\Delta h_{12-1}} = (H_1^0 - H_{12}) - \Delta h_{12-1}$

  $v_{\Delta h_{12-2}} = dH_2 + f_{\Delta h_{12-2}}, \;\; f_{\Delta h_{12-2}} = (H_2^0 - H_{12}) - \Delta h_{12-2}$

  $v_{\Delta h_{2-1}} = dH_1 - dH_2 + f_{\Delta h_{2-1}}, \;\; f_{\Delta h_{2-1}} = (H_1^0 - H_2^0) - \Delta h_{2-1}$

  $v_{\Delta h_{3-2}} = dH_2 - dH_3 + f_{\Delta h_{3-2}}, \;\; f_{\Delta h_{2-1}} = (H_2^0 - H_3^0) - \Delta h_{3-2}$

Funkcionalni model

- **Primer 2 – jednodimenzionalna geodetska mreža**

Formiranje matrice dizajna $\mathbf{A}$ i vektora slobodnih članova $\mathbf{f}$:

$$\mathbf{A} = \begin{array}{c} \begin{matrix} dH_1 & dH_2 & dH_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \end{array} \begin{matrix} \Delta h_{10-1} \\ \Delta h_{10-3} \\ \Delta h_{11-2} \\ \Delta h_{11-3} \\ \Delta h_{12-1} \\ \Delta h_{12-2} \\ \Delta h_{2-1} \\ \Delta h_{3-2} \end{matrix} \qquad \mathbf{f} = \begin{bmatrix} f_{\Delta h_{10-1}} \\ f_{\Delta h_{10-3}} \\ f_{\Delta h_{11-2}} \\ f_{\Delta h_{11-3}} \\ f_{\Delta h_{12-1}} \\ f_{\Delta h_{12-2}} \\ f_{\Delta h_{2-1}} \\ f_{\Delta h_{3-2}} \end{bmatrix}$$

Napomena:

- Slobodna članove izraziti u milimetrima.

Stohastički model

- **Primer 2 – jednodimenzionalna geodetska mreža**

Formiranje težina merenja:

$$P_{\Delta h_{i-j}} = \frac{1}{S_{i-j}},$$

S_{i-j} – dužina nivelmanske strane u kilometrima.

$$P_{\Delta h_{10-1}} = \frac{1}{S_{10-1}}, P_{\Delta h_{12-1}} = \frac{1}{S_{12-1}}$$

$$P_{\Delta h_{10-3}} = \frac{1}{S_{10-3}}, P_{\Delta h_{12-2}} = \frac{1}{S_{12-2}}$$

$$P_{\Delta h_{11-2}} = \frac{1}{S_{11-2}}, P_{\Delta h_{2-1}} = \frac{1}{S_{2-1}}$$

$$P_{\Delta h_{11-3}} = \frac{1}{S_{11-3}}, P_{\Delta h_{3-2}} = \frac{1}{S_{3-2}}$$

$$\mathbf{P} = \begin{bmatrix} P_{\Delta h_{10-1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{\Delta h_{10-3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{\Delta h_{11-2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{\Delta h_{11-3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{\Delta h_{12-1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{12-2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{2-1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{\Delta h_{3-2}} \end{bmatrix} \begin{matrix} \Delta h_{10-1} \\ \Delta h_{10-3} \\ \Delta h_{11-2} \\ \Delta h_{11-3} \\ \Delta h_{12-1} \\ \Delta h_{12-2} \\ \Delta h_{2-1} \\ \Delta h_{3-2} \end{matrix}$$

Kompletno rešenje primera 2 dostupno je u fajlu *Račun izravnanja - Vežba 5.xlsx*.

Primena metoda najmanjih kvadrata (MNK)

- Sistem normalnih jednačina

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$$\mathbf{N} = \mathbf{A}^T\mathbf{P}\mathbf{A}$$
$$\mathbf{n} = \mathbf{A}^T\mathbf{P}\mathbf{f}$$

- Ocena nepoznatih parametara i popravaka merenih veličina

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Kontrola računanja:
$\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}} \equiv \mathbf{f}^T\mathbf{P}\mathbf{f} + \mathbf{n}^T\hat{\mathbf{x}}$

- Ocena disperzionog koeficijenta

$$m_0^2 = \frac{\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}}{f}$$

$f = n - u,\ n$ – broj merenja, u – broj nepoznatih parametara

Globlani test na grube greške

- Hipoteze

$H_0: \sigma^2 = \sigma_0^2$ protiv $H_a: \sigma^2 \neq \sigma_0^2$,

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Izravnate vrednosti merenja i nepoznatih parametara

- Izravnate vrednosti nepoznatih parametara

$$\hat{H}_1 = H_1^0 + \widehat{dH}_1$$
$$\hat{H}_2 = H_2^0 + \widehat{dH}_2$$
$$\hat{H}_3 = H_3^0 + \widehat{dH}_3$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \widehat{dH}_1 \\ \widehat{dH}_2 \\ \widehat{dH}_3 \end{bmatrix}$$

- Izravnate vrednosti merenih veličina

$$\widehat{\Delta h}_{10-1} = \Delta h_{10-1} + \hat{v}_{\Delta h_{10-1}}$$
$$\widehat{\Delta h}_{10-3} = \Delta h_{10-3} + \hat{v}_{\Delta h_{10-3}}$$
$$\widehat{\Delta h}_{11-2} = \Delta h_{11-2} + \hat{v}_{\Delta h_{11-2}}$$
$$\widehat{\Delta h}_{11-3} = \Delta h_{11-3} + \hat{v}_{\Delta h_{11-3}}$$
$$\widehat{\Delta h}_{12-1} = \Delta h_{12-1} + \hat{v}_{\Delta h_{12-1}}$$
$$\widehat{\Delta h}_{12-2} = \Delta h_{12-2} + \hat{v}_{\Delta h_{12-2}}$$
$$\widehat{\Delta h}_{2-1} = \Delta h_{2-1} + \hat{v}_{\Delta h_{2-1}}$$
$$\widehat{\Delta h}_{3-2} = \Delta h_{3-2} + \hat{v}_{\Delta h_{3-2}}$$

$$\hat{\mathbf{v}} = \begin{bmatrix} \hat{v}_{\Delta h_{10-1}} \\ \hat{v}_{\Delta h_{10-3}} \\ \hat{v}_{\Delta h_{11-2}} \\ \hat{v}_{\Delta h_{11-3}} \\ \hat{v}_{\Delta h_{12-1}} \\ \hat{v}_{\Delta h_{12-2}} \\ \hat{v}_{\Delta h_{2-1}} \\ \hat{v}_{\Delta h_{3-2}} \end{bmatrix}$$

Analiza tačnosti geodetskih mreža

- *A posteriori* standardna devijacija (globalna mera tačnosti)

$$m_0 = \sqrt{\frac{\hat{\mathbf{v}}^T \mathbf{P} \hat{\mathbf{v}}}{f}},$$

pri čemu je $f = n - u$ broj stepeni slobode.

- Standardne devijacije visina repera (lokalne mere tačnosti)

$$\hat{\sigma}_{H_i} = m_0 \sqrt{Q_{\hat{H}_i}}$$

σ_0 - *a priori* standardna devijacija

$Q_{\hat{H}_i}$ - dijagonalni elementi kofaktorske matrice $\mathbf{Q}_{\hat{\mathbf{x}}}$

$$\mathbf{Q}_{\hat{\mathbf{x}}} = \begin{bmatrix} Q_{\hat{H}_1} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & Q_{\hat{H}_m} \end{bmatrix}$$